

# Eukaryotic cell polarity and protein sorting

Andrea Gamba



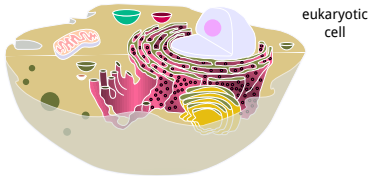
**POLITECNICO  
DI TORINO**

Landau Institute, April 27, 2018

## Plan of the talk

- ◆ Membrane identity in eukaryotic cells
- ◆ How membrane identity is created and maintained
- ◆ Molecular sorting
- ◆ Phenomenological theory
- ◆ Experimental validation

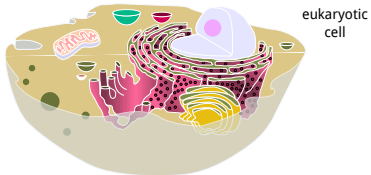
## The eukaryotic cell




eukaryotic  
cell

$R \sim 10 \mu\text{m}$  →

## The eukaryotic cell

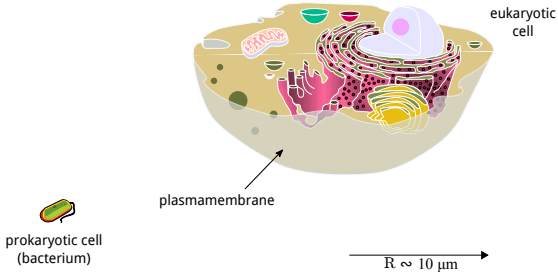


prokaryotic cell  
(bacterium)

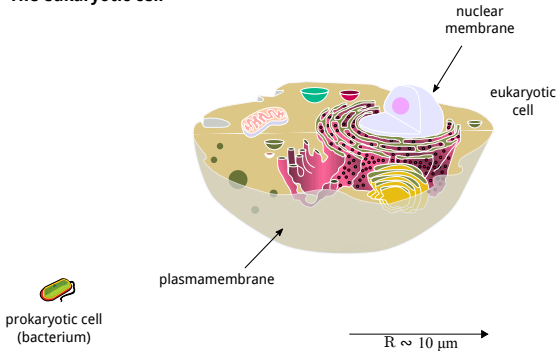
$R \sim 10 \mu\text{m}$  



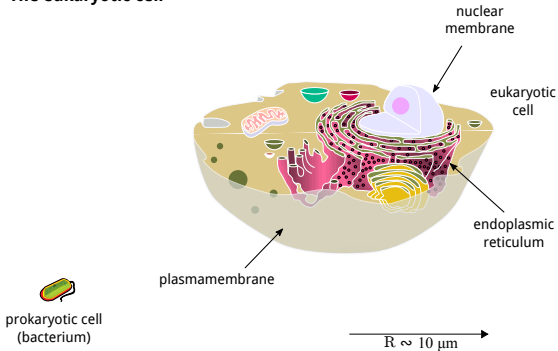
## The eukaryotic cell



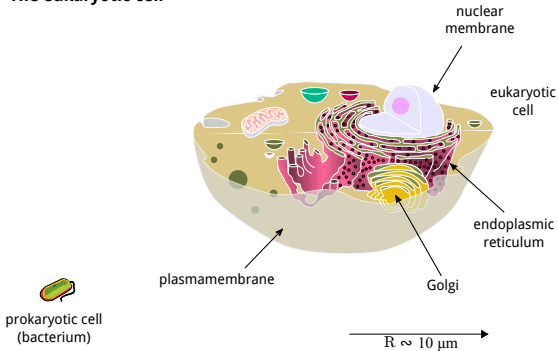
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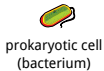
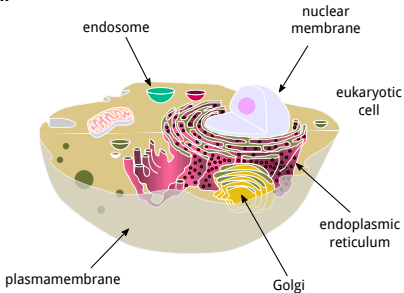
## The eukaryotic cell



## The eukaryotic cell

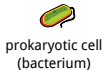
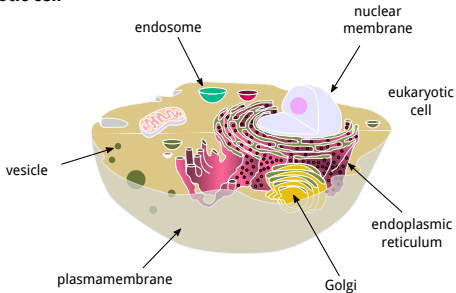


## The eukaryotic cell



$R \sim 10 \mu\text{m}$

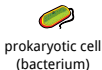
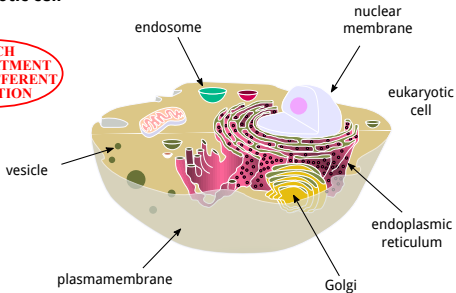
## The eukaryotic cell



$R \sim 10 \mu\text{m}$

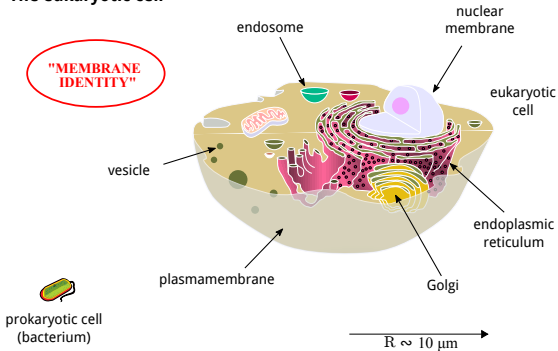
# The eukaryotic cell

**EACH  
COMPARTMENT  
HAS A DIFFERENT  
FUNCTION**



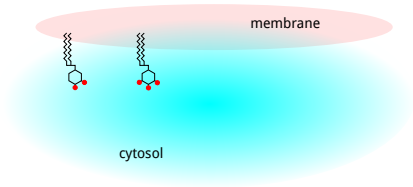
$R \sim 10 \mu\text{m}$

## The eukaryotic cell

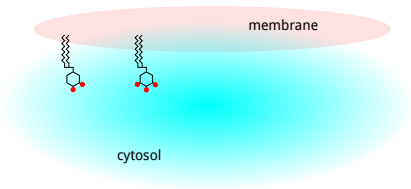
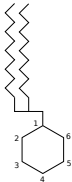




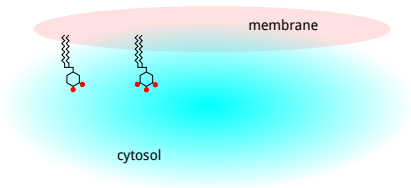
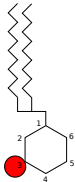
## Signaling molecules



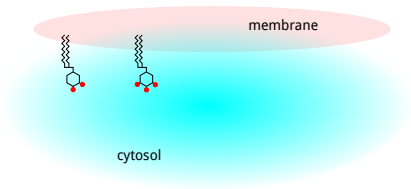
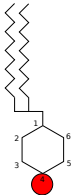
## Signaling molecules



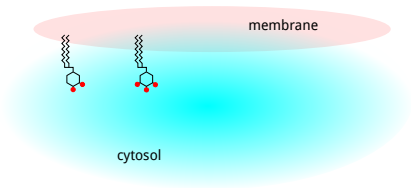
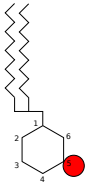
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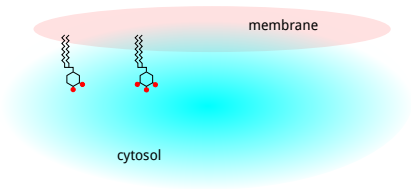
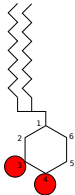
## Signaling molecules



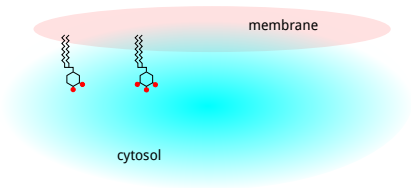
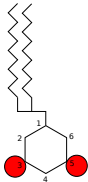
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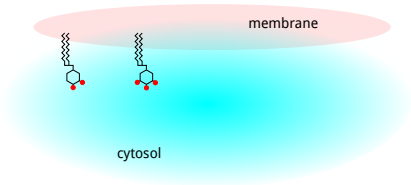
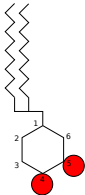
## Signaling molecules



## Signaling molecules

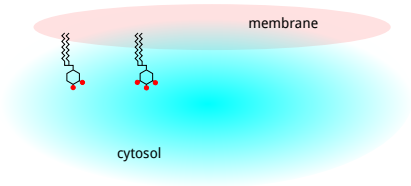
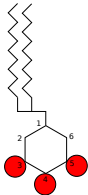


## Signaling molecules

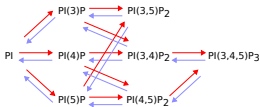
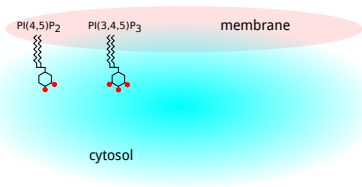
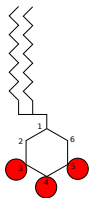




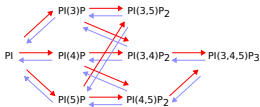
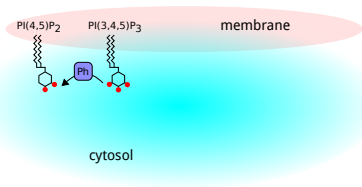
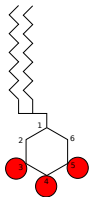
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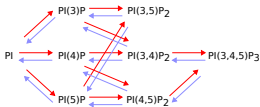
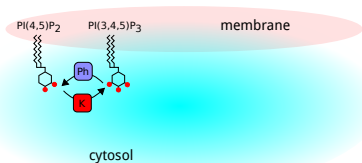
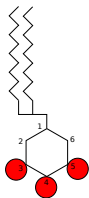
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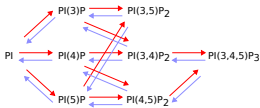
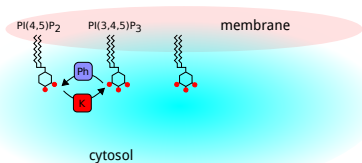
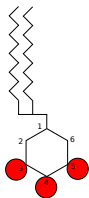
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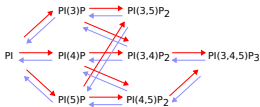
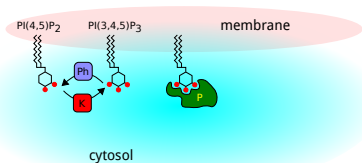
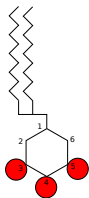
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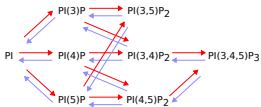
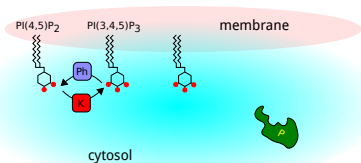
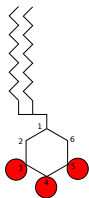
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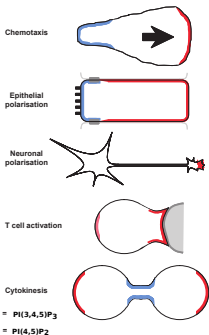
## Signaling molecules



## Signaling molecules



## Signaling domains





## Signaling domains

Chemotaxis



Epithelial polarisation



Neuronal polarisation



T cell activation



Cytokinesis



 = PI(3,4,5)P<sub>3</sub>

 = PI(4,5)P<sub>2</sub>

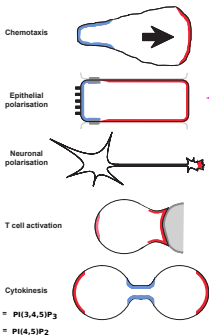


Linked  
image  
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Devreotes, Janetopoulos  
J. Biol. Chem. 2003.

Leslie et al. Oncogene, 2008

## Signaling domains



  
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Martin-Belmonte et al.  
Cell 2007

Leslie et al. Oncogene, 2008

## Signaling domains

Chemotaxis



Epithelial polarisation



Neuronal polarisation



T cell activation



Cytokinesis



 = PI(3,4,5)P<sub>3</sub>

 = PI(4,5)P<sub>2</sub>

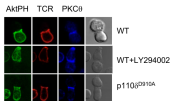
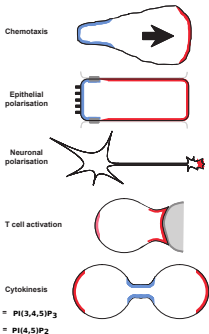


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Menager et al.  
J. Neurochem. 2004.

Leslie et al. Oncogene, 2008

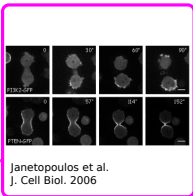
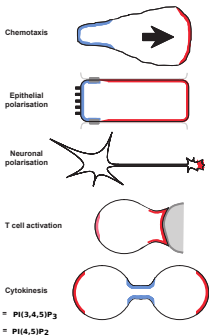
# Signaling domains



Garcon et al.  
Blood 2007

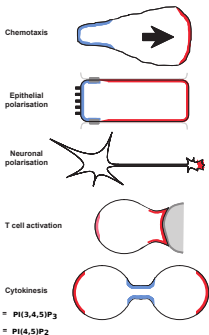
Leslie et al. Oncogene, 2008

## Signaling domains

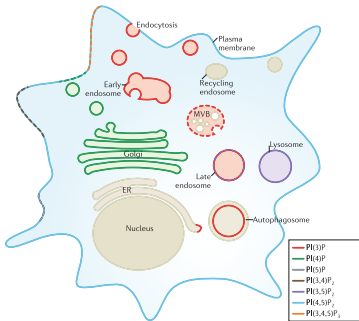


Leslie et al. Oncogene, 2008

## Signaling domains



# Membrane identity



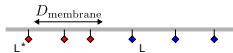
modified from Jean & Kiger  
Nat Rev Mol Cell Biol 2012

## Domain formation

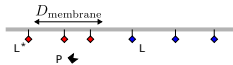




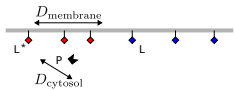
## Domain formation



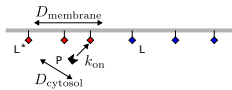
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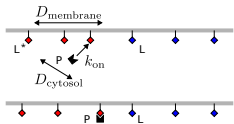
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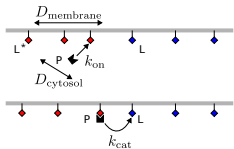
## Domain formation



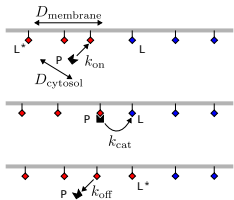
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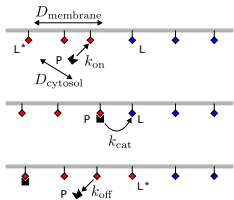
## Domain formation



## Domain formation

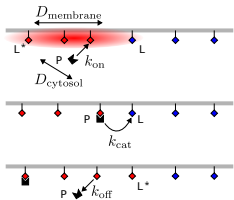


## Domain formation

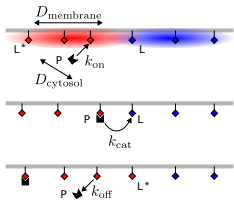




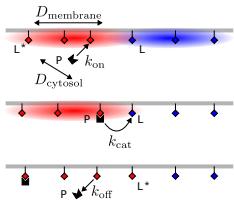
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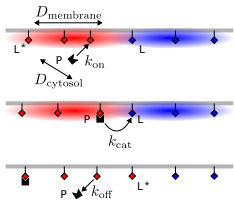
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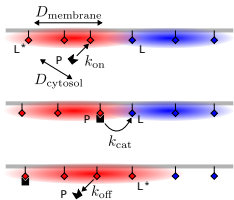
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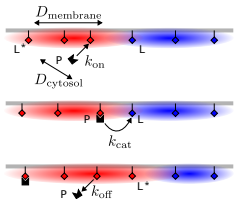
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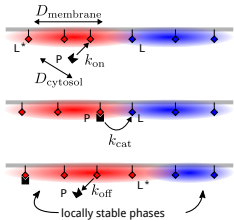
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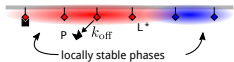
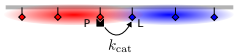
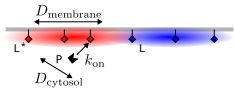
## Domain formation



## Domain formation



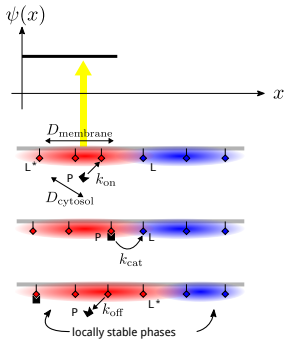
## Domain formation



P depleted from cytosol as red phase grows

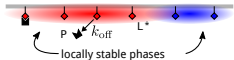
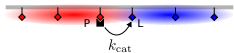
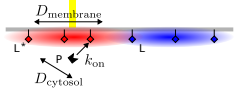


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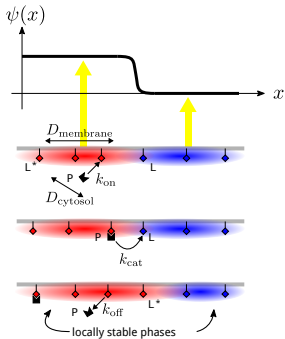
P depleted from cytosol as red phase grows

## Domain formation

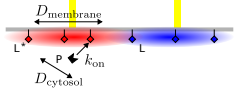
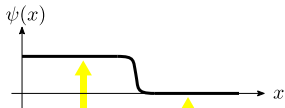


P depleted from cytosol as red phase grows

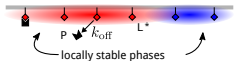
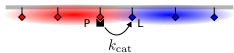
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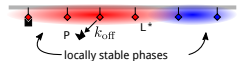
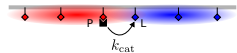
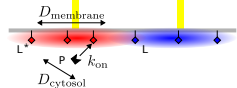
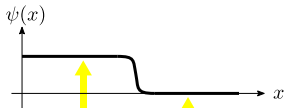


$$\frac{\partial \psi}{\partial t} = -D \nabla^2 \psi + V'(\epsilon, \psi) + \xi$$

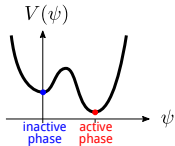


P depleted from cytosol as red phase grows

## Domain formation

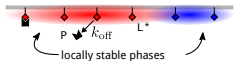
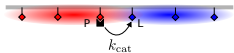
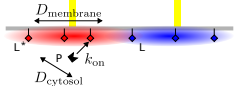
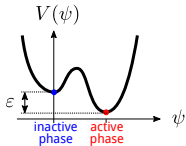
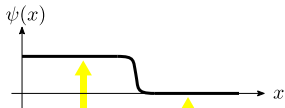


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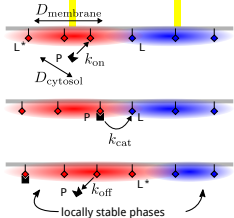
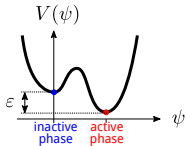
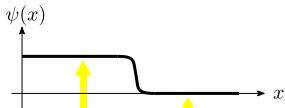
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P depleted from cytosol as red phase grows

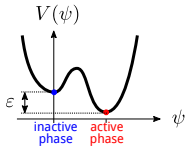
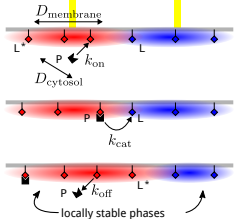
## Domain formation



$$\begin{aligned} \frac{\partial \psi}{\partial t} &= -D \nabla^2 \psi + V'(\epsilon, \psi) + \xi \\ &= -\frac{\delta \mathcal{F}}{\delta \psi} + \xi \end{aligned}$$

P depleted from cytosol as red phase grows

## Domain formation



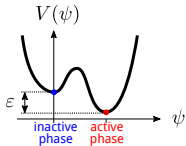
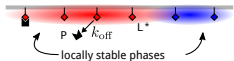
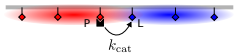
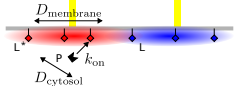
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$$\mathcal{F} = \int \left[ \frac{D}{2} (\nabla \psi)^2 + V(\epsilon, \psi) \right] dA$$

P depleted from cytosol as red phase grows



## Domain formation

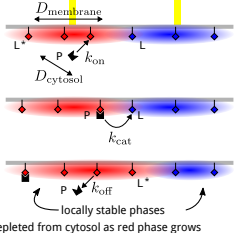


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$$\epsilon \propto 1 - \frac{1}{A_{\text{eq}}} \int \psi dA$$

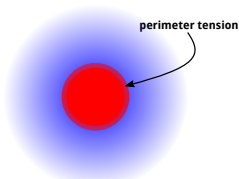
P depleted from cytosol as red phase grows

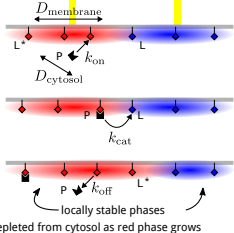


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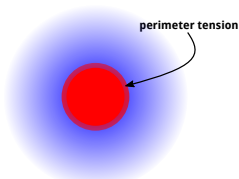
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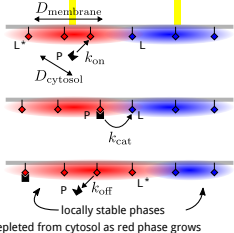
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- nucleation
- coarsening
- coalescence

AG, I Kolokolov,  
V Lebedev, G Ortenzi  
PRL 2007





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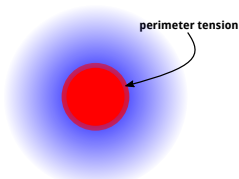
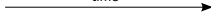
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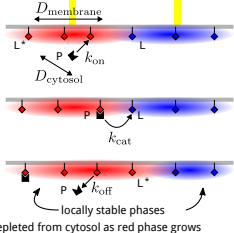
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PRL 2007

time



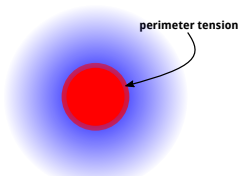


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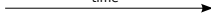
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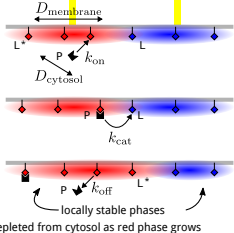


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PRL 2007

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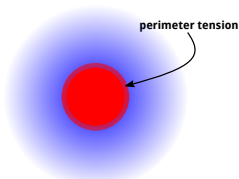




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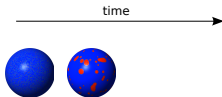
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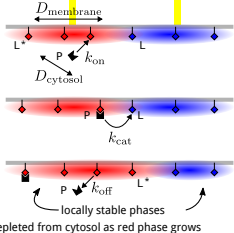
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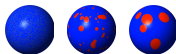
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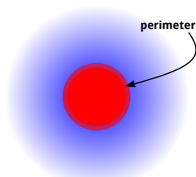
AG, I Kolokolov,  
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PRL 2007

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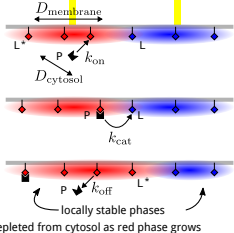


perimeter tension



locally stable phases

P depleted from cytosol as red phase grows

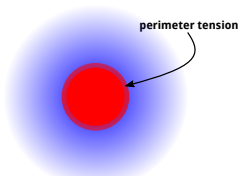


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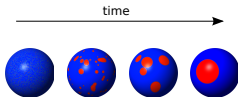
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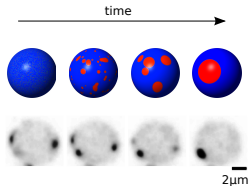
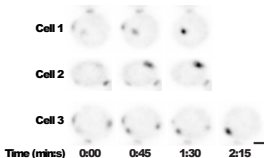
AG, I Kolokolov,  
V Lebedev, G Ortenzi  
PRL 2007





## Domain coarsening

The competition hypothesis predicts that polarity establishment should frequently proceed via a transient intermediate stage with **more than one polarity cluster**, but there is limited experimental evidence for such intermediates, as only rare, fleeting two-cluster instances were identified in *rsr1Δ* cells (Howell et al., 2009). Thus, either competition occurs very rapidly, or some other mechanism ensures that only a single cluster develops. To distinguish between these possibilities, we developed **higher-resolution filming conditions** that circumvented the phototoxicity of previous protocols. We now document the frequent formation of more than one polarity cluster, and **rapid competition between clusters**, during symmetry-breaking polarization in *rsr1Δ* cells. Rapid filming of initial polarity



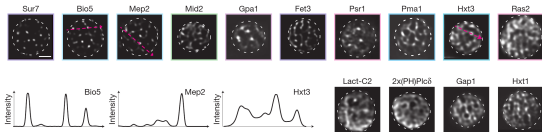
**Figure 1. Dynamic Behaviors of Bem1p-GFP during Polarity Establishment**

Inverted images (so dark spots represent concentrations of Bem1p-GFP) from movies of cells breaking symmetry. Time in min:s. Scale bar, 2 μm. (Nack) The "old" neck signal in the attached daughter cell.

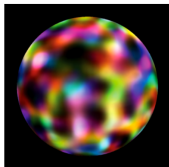
(A) Growth of multiple Bem1p clusters (numbered in the key at right) and resolution to a single cluster.  $t = 0$  indicates the first detection of polarized signal.



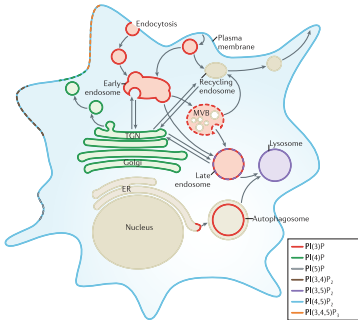
## Microdomains



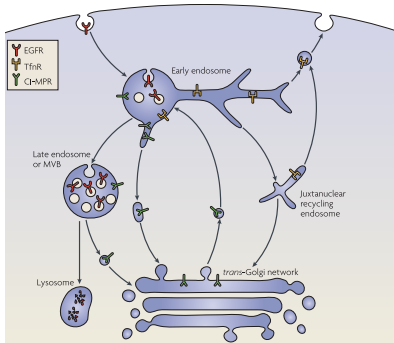
Spira et al. Pathwork organization of the yeast plasma membrane into numerous coexisting domains, Nat Cell Biol 14 (2012)



## A dynamic picture

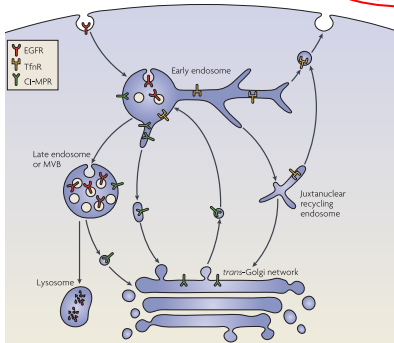


## Molecular sorting



## Molecular sorting

**SORTING  
COUNTERS DIFFUSION,  
ORDERS THE  
SYSTEM**



## **Molecular sorting**

Underlying physical mechanism?

## Molecular sorting

Underlying physical mechanism?

coupling:



## Molecular sorting

Underlying physical mechanism?

coupling:

**affinity-driven aggregation**

## Molecular sorting

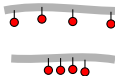


Underlying physical mechanism?

coupling:

**affinity-driven aggregation**

## Molecular sorting

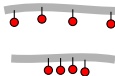


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## Molecular sorting

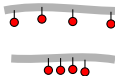


Underlying physical mechanism?

coupling:

**affinity-driven aggregation** +

## Molecular sorting



Underlying physical mechanism?

coupling:

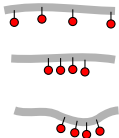
**affinity-driven aggregation** + **vesicle nucleation**

## Molecular sorting

Underlying physical mechanism?

coupling:

**affinity-driven aggregation** + **vesicle nucleation**



## Molecular sorting

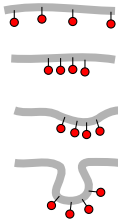
Underlying physical mechanism?

coupling:

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## Molecular sorting

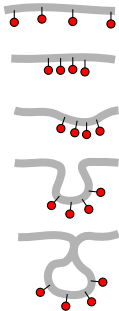
Underlying physical mechanism?

coupling:

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## Molecular sorting

Underlying physical mechanism?

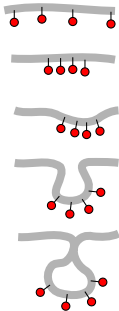
coupling:

**affinity-driven aggregation**

+

**vesicle nucleation**

should result in:



## Molecular sorting

Underlying physical mechanism?

coupling:

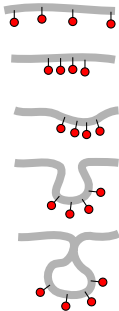
**affinity-driven aggregation**

+

**vesicle nucleation**

should result in:

**spontaneous distillation  
of molecular factors**



## Molecular sorting

Underlying physical mechanism?

coupling:

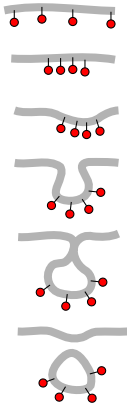
**affinity-driven aggregation**

+

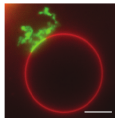
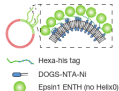
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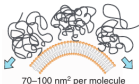
**spontaneous distillation  
of molecular factors**



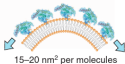
# Molecular crowding induces membrane bending and vesicle nucleation



Large IDP domains  
drive bending more efficiently



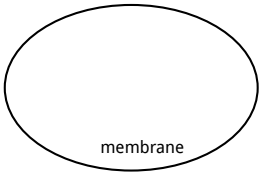
Small globular domains  
drive bending less efficiently



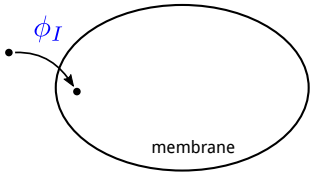
JC Stachowiak et al  
Membrane bending  
by protein-protein crowding  
Nat Cell Biol 2012

DJ Busch et al  
Intrinsically disordered proteins drive  
membrane curvature.  
Nat Commun 2015

## Minimal model of molecule sorting

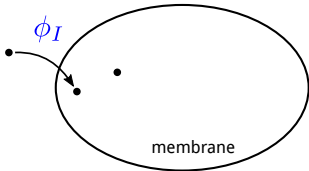


## Minimal model of molecule sorting



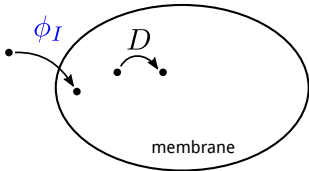
$\phi_I$  : incoming molecule flux

## Minimal model of molecule sorting



$\phi_I$  : incoming molecule flux

## Minimal model of molecule sorting

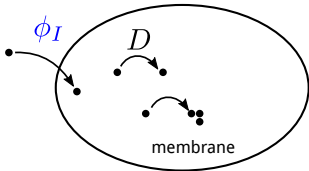


$\phi_I$  : incoming molecule flux

$D$  : diffusivity



## Minimal model of molecule sorting

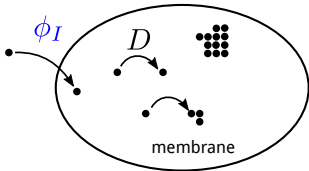


$\phi_I$  : incoming molecule flux

$D$  : diffusivity

attractive interaction

## Minimal model of molecule sorting

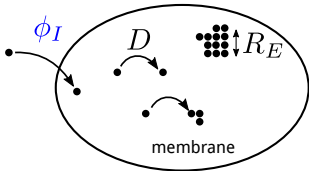


$\phi_I$  : incoming molecule flux

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attractive interaction

## Minimal model of molecule sorting



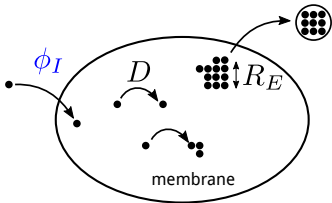
$\phi_I$  : incoming molecule flux

$D$  : diffusivity

$R_E$  : extraction size

attractive interaction

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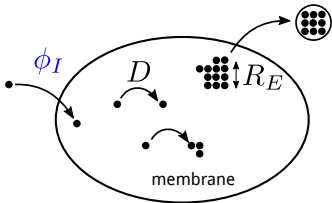
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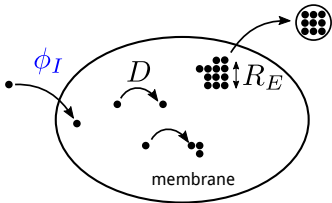
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Sorting  
efficiency?

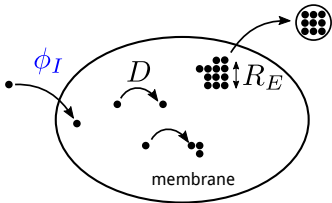
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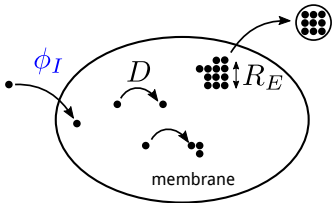
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Sorting efficiency?

$\bar{T}$  : average time spent by a molecule in the system

$S = \frac{1}{\bar{T}}$  : sorting rate



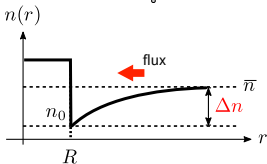
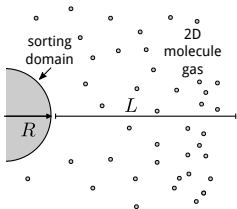
## Domain growth

density profile of  
freely diffusing molecules:

$$n(r) = n_0 + \frac{\log r/R}{\log L/R} \Delta n$$

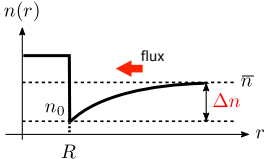
flux of molecules  
towards sorting domain:

$$\Phi_R = 2\pi r D \left. \frac{\partial n}{\partial r} \right|_{r=R} = \frac{2\pi D \Delta n}{\ln L/R}$$



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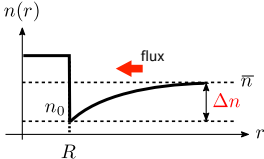
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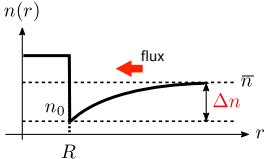
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$N(R, t)$  : number distribution of domains



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$N(R, t)$  : number distribution of domains

$$\frac{\partial}{\partial t} N + \frac{\partial}{\partial R} \left( \frac{\Phi_R}{2\pi R} N \right) = \gamma(R) N$$

$\gamma(R)$  : extraction rate



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in a stationary condition:

$$N_{\text{st}}(R) = \frac{JR \ln L/R}{D\Delta n} \exp \left[ - \int_0^R \frac{r \ln L/r}{D\Delta n} \gamma(r) dr \right]$$

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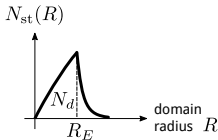
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the total number of domains is found self-consistently:

$$\int \Phi_R N_{\text{st}}(R) dR = \phi_I \Rightarrow J \sim \frac{\phi_I}{R_E^2}$$

$\phi_I$  : incoming molecule flux

$$N_d = \int N_{\text{st}}(R) dR \sim \frac{\phi_I}{D \Delta n}$$

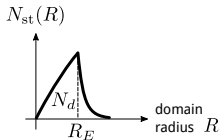


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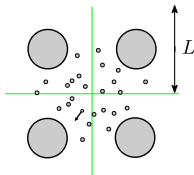
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Average time spent on the membrane:

$$\bar{T} = \bar{T}_f + \bar{T}_d$$

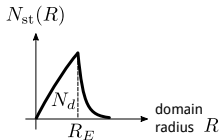


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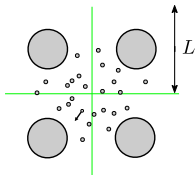
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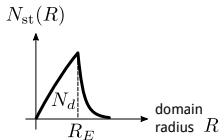


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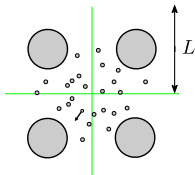
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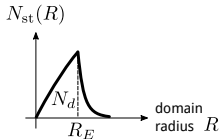


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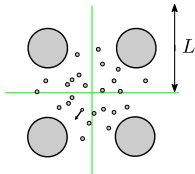
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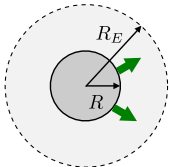
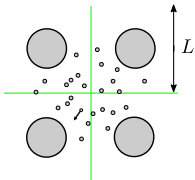
$$\bar{T}_f \sim \frac{L^2}{D} \sim \frac{1}{DN_d} \sim \frac{\Delta n}{\phi_I}$$



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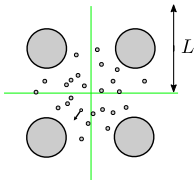
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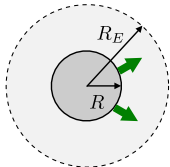
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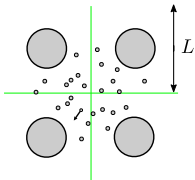
$$\bar{T}_d \sim \frac{R_E^2}{\Phi_R}$$



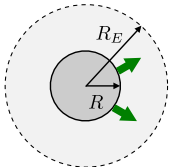
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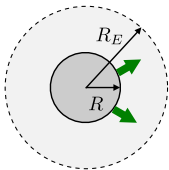
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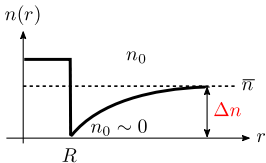
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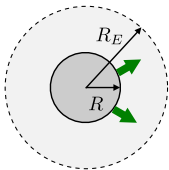
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For absorbing domains  $\Delta n \sim \bar{n}$

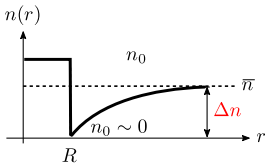


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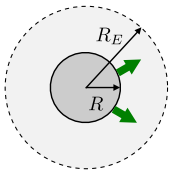


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$$\frac{dN_{d,\text{new}}}{dt}$$

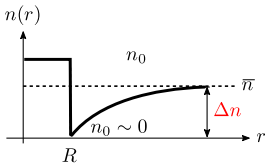


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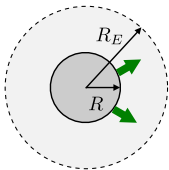
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$$\frac{dN_{d,\text{new}}}{dt} = CD\bar{n}^2$$



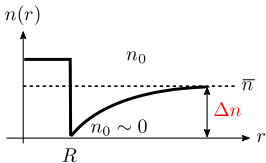


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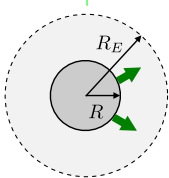


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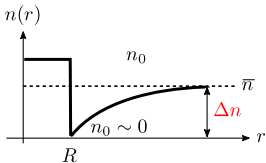
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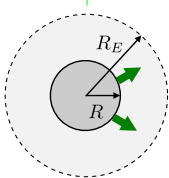
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$$\Rightarrow \Delta n \sim \bar{n} \sim \left( \frac{\phi_I}{CDR_E^2} \right)^{1/2}$$



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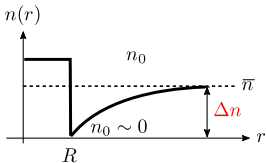


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$C$  ~ aggregation strength



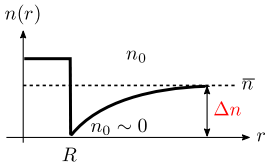
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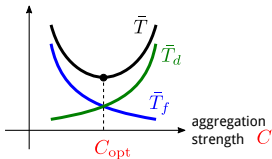
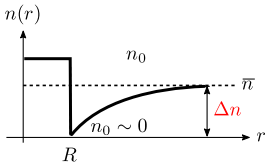
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$$C_{\text{opt}} \sim R_E^{-4}$$



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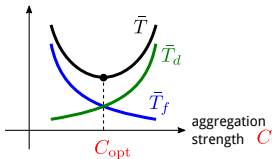
$$\bar{T}_d \sim C^{1/2} \frac{R_E^3}{(D\phi_I)^{1/2}}$$

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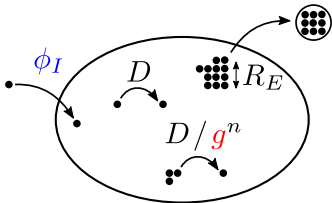
$$\bar{T}_f^{\text{opt}} \sim \bar{T}_d^{\text{opt}} \sim \frac{R_E}{(D\phi_I)^{1/2}}$$

$$\bar{n}^{\text{opt}} \sim \Delta n^{\text{opt}} \sim \frac{\phi_I^{1/2} R_E}{D^{1/2}}$$

$$\rho^{\text{opt}} \sim \bar{n}^{\text{opt}} \text{ is also minimal at fixed } \phi_I \quad (\bar{T} = \rho \phi_I)$$



## Numerical simulations



$\phi_I$  : incoming molecule flux

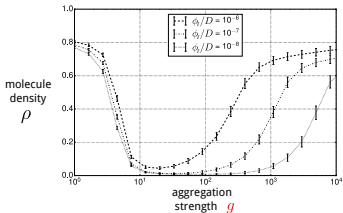
$D$  : diffusivity

$g$  : aggregation strength

$n$  : number of neighbours

$R_E$  : extraction size

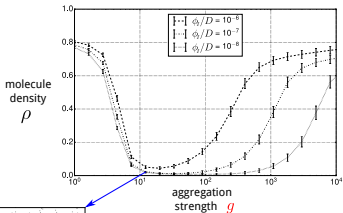
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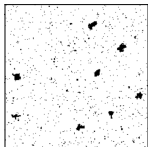
$$\bar{T} = \rho \phi_I$$



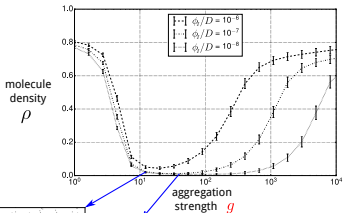
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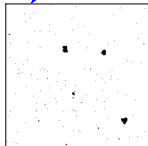
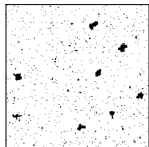
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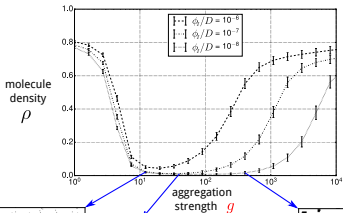
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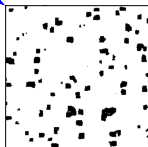
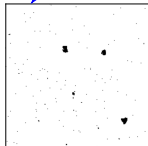
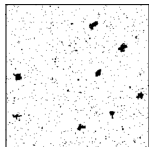
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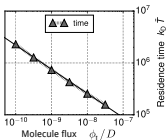
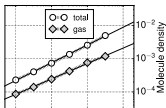
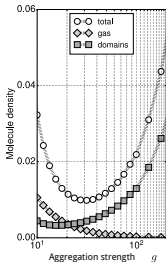
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## Numerical simulations

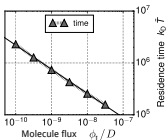
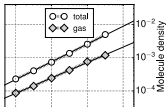
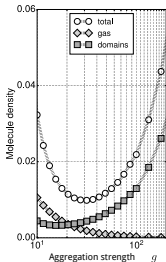


$$\rho \propto \phi_I^{0.53}$$

$$\bar{n} \propto \phi_I^{0.46}$$

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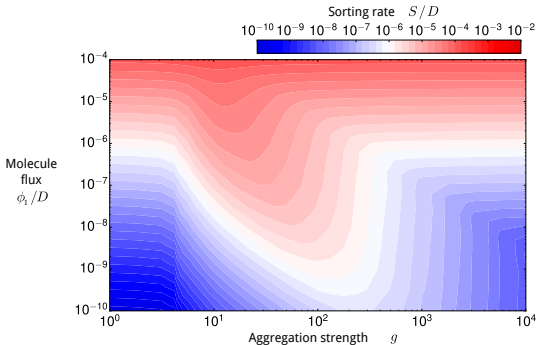


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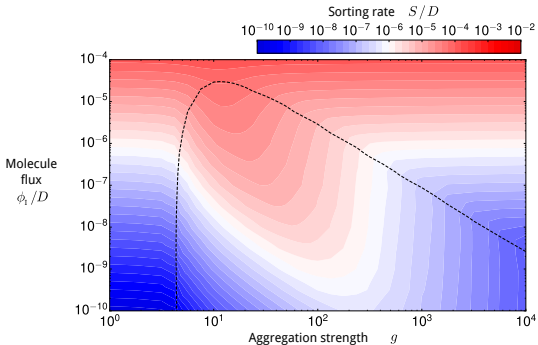
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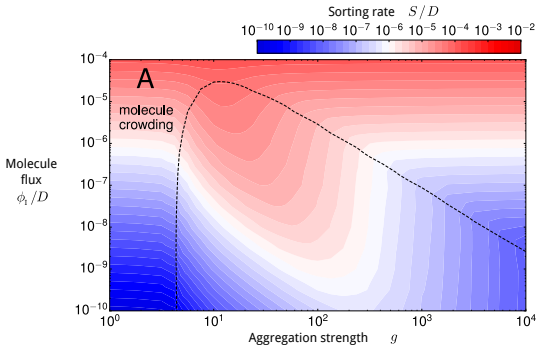
## A phase diagram of sorting



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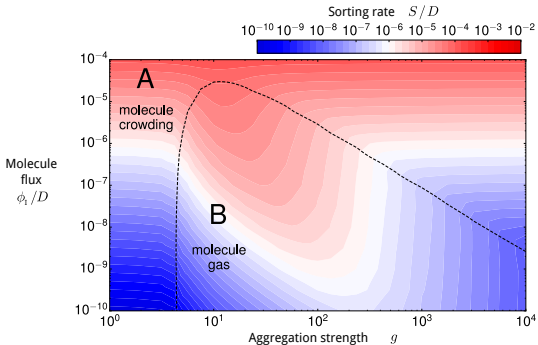


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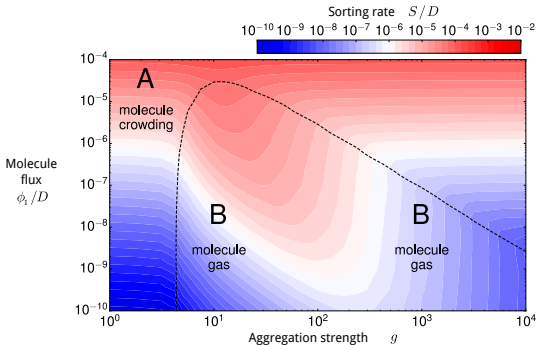




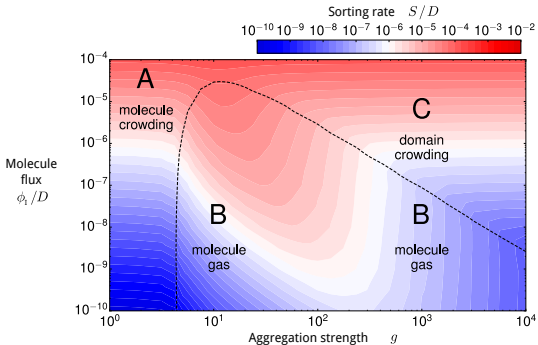
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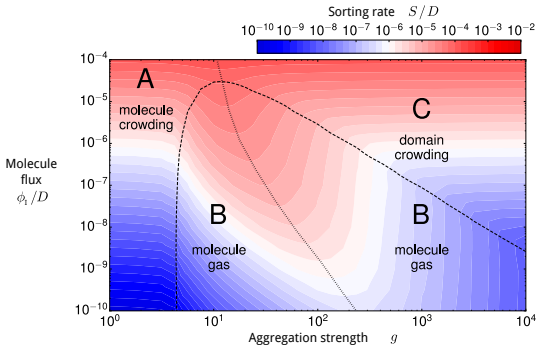
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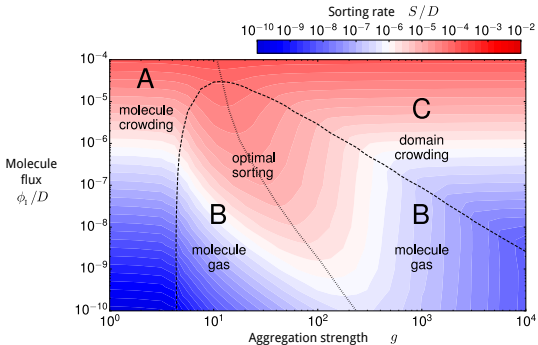
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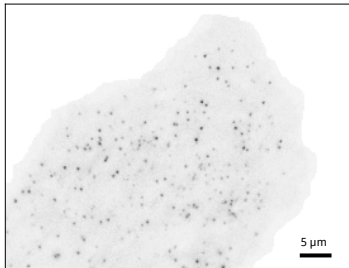
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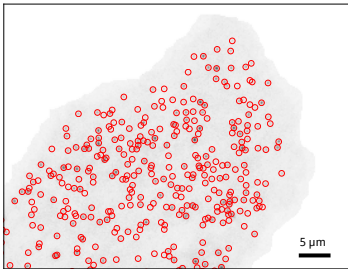
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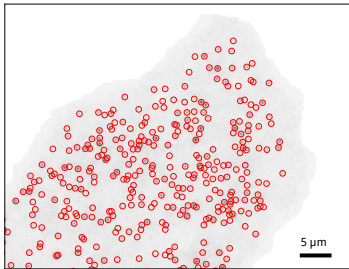
## Endocytic sorting



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90 s



100 s



110 s



120 s



130 s



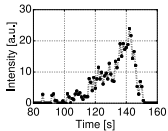
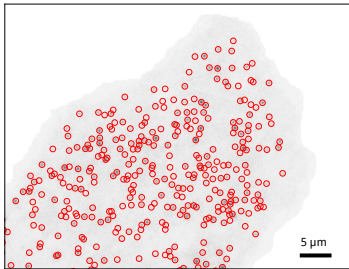
140 s



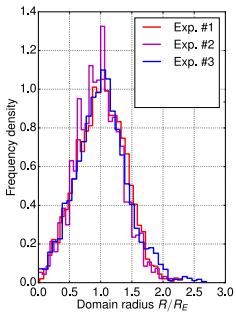
150 s



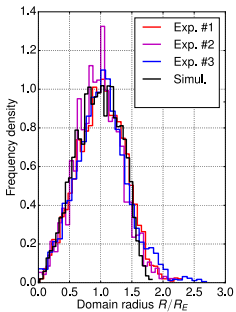
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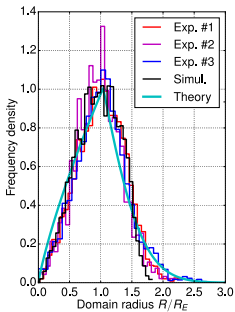
## Distribution of domain sizes



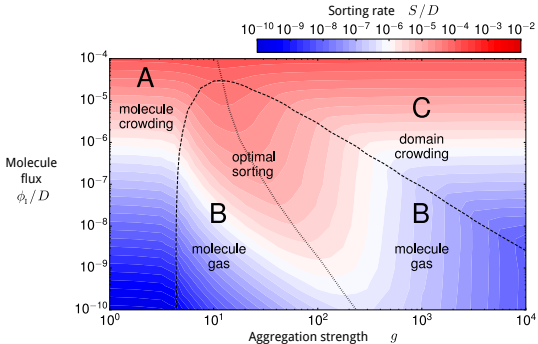
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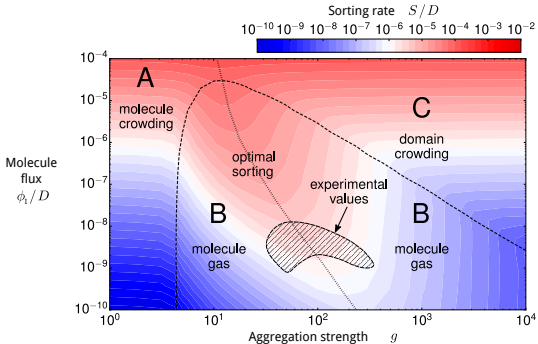
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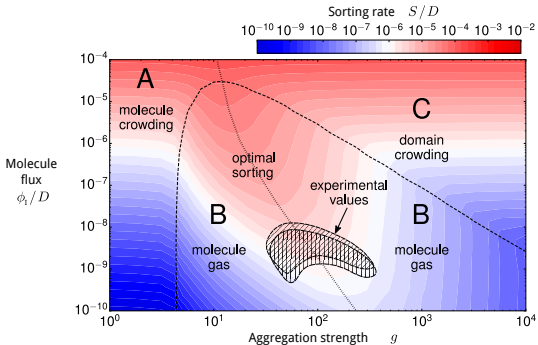
## Do cells live close to the optimal regime?



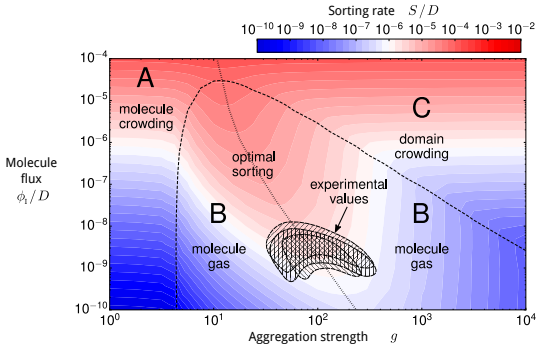
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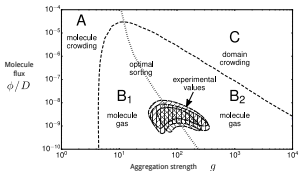
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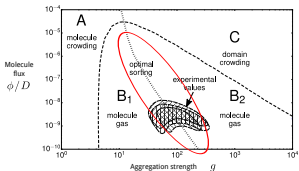
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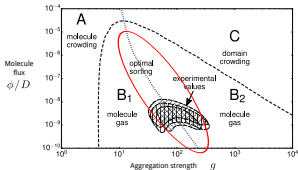
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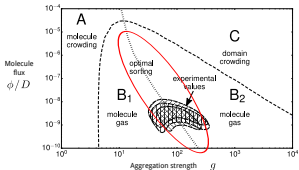
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## Collaboration

### ◆ Candiolo Cancer Institute

- Guido Serini
- Donatella Valdembri



### ◆ Landau Institute for Theoretical Physics, Moscow

- Igor Kolokolov
- Vladimir Lebedev



### ◆ Politecnico di Torino

- Marco Zamparo
- Luca Dall'Asta

