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DUALITY IN POWER-LAW LOCALIZATION IN DISORDERED ONE-DIMENSIONAL SYSTEMS

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Levitov's paradigm (1990)

$$H = \sum_{\vec{n}} \varepsilon_{\vec{n}} \, c^{+}_{\vec{n}} c_{\vec{n}} - \sum_{\vec{n},\vec{m}} V_{\vec{n}-\vec{m}} \, c^{+}_{\vec{n}} c_{\vec{m}}$$

Disorder

Dipole hopping term

$V(\vec{r}) \propto r^{-3}$, in d=3 dim. space

angular average $\overline{V(r,\theta)} = 0$,

Anderson localization is destroyed

Mirlin-Fyodorov RMT (1996)

$$\langle H_{nn}^2 \rangle = 1, \ \langle H_{n\neq m}^2 \rangle = \left(\frac{b}{|n-m|} \right)^{2\gamma}$$

$$\langle H_{nm} \rangle = 0$$

$$H = \sum_n \varepsilon_n \ c_n^+ c_n - \sum_{n,m} V_{nm} \ c_n^+ c_m$$

$$\varepsilon_n = H_{nn} \quad d = 1 \quad V_{nm} = H_{n\neq m}$$

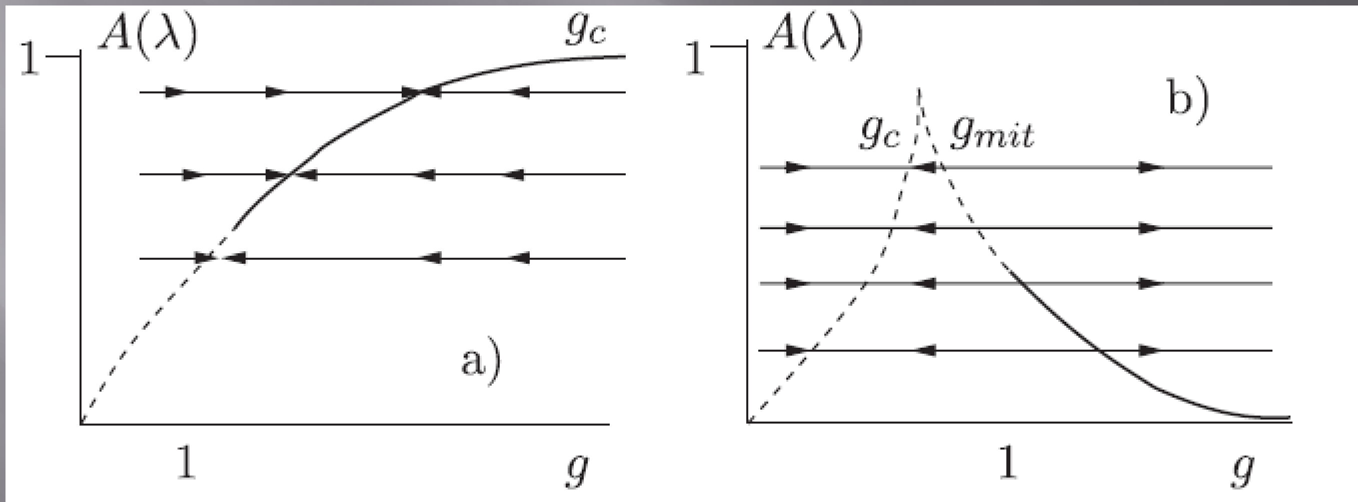
γ>1, PL localization

γ=1, multifractal critical states

γ< 1, delocalized ergodic states

Altshuler-Aleiner-Efetov 2D dipole 1/r^2 hopping (2011)

No localization for 2D dipole hopping



GOE

GUE

Intermediate conclusion

Delocalized states for disordered systems with long-range hopping
V(r)~r^{–γ}, γ<= d

$V(r) \sim r^{-\gamma}, \quad \gamma \le d$

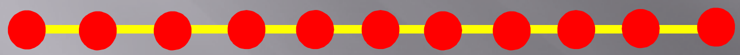
Valid if the hopping averages to zero upon either ensemble or angular averaging

What about sign-definite, e.g. deterministic long-range hopping?

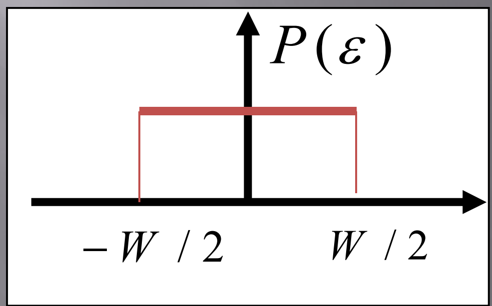
Deterministic PL and Euclidean PL hopping models

$$H = \sum_n \varepsilon_n \; c_n^+ c_n - \sum_{n,m} V_{nm} \; c_n^+ c_m$$

DPL



Regular lattice



$$V_{nm} = \frac{1}{|n-m|^{\gamma}}$$

EPL

Randomly positioned sites

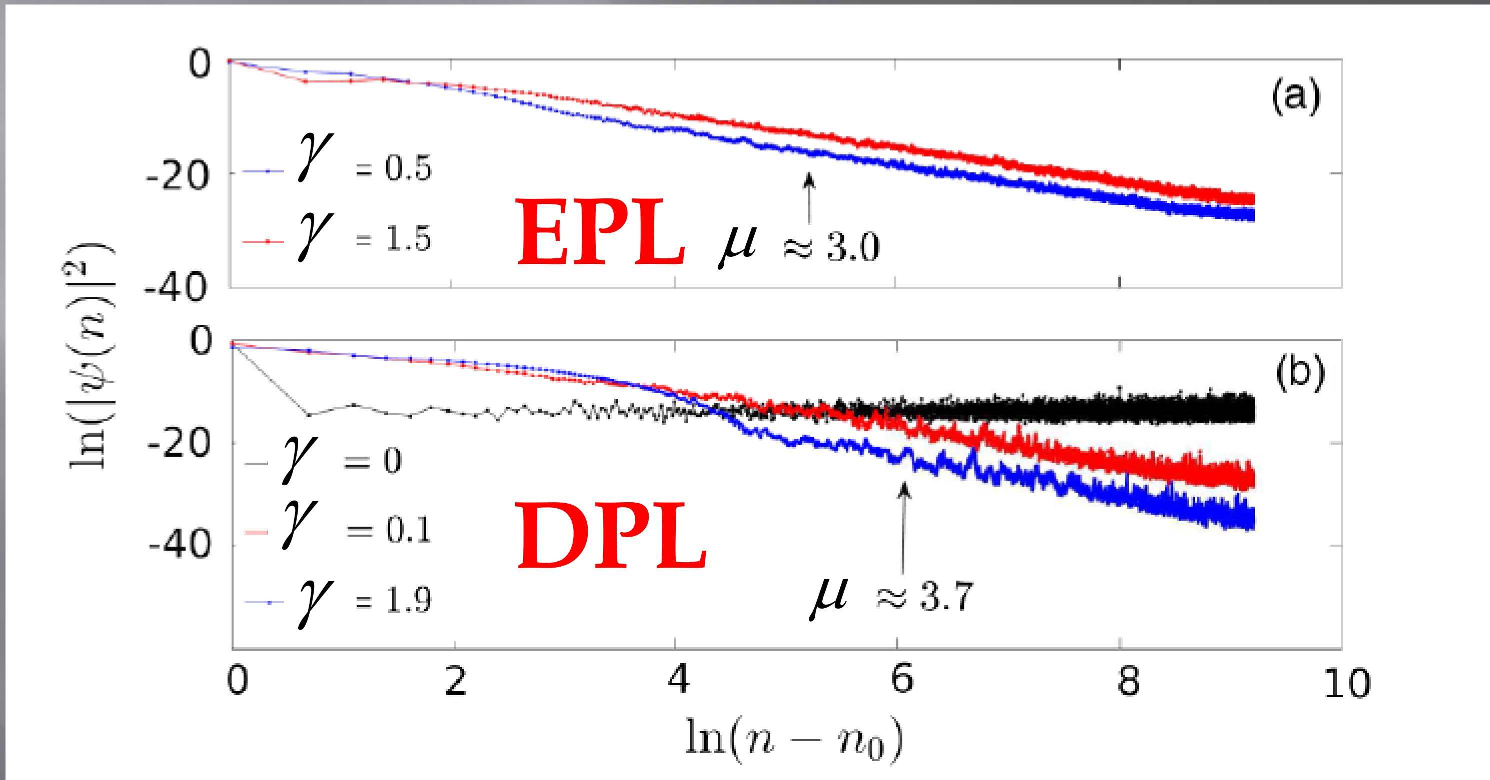
$$\varepsilon = 0$$

$$V_{nm} = \frac{1}{|r(n)-r(m)|^{\gamma}}$$

Power-law localization and duality in DPL and EPL models

$$\exp[\langle \ln |\Psi|^2 \rangle] = \frac{c}{|i - i_0|^{\mu}}$$

$$V_{nm} = \frac{1}{|n-m|^{\gamma}}$$

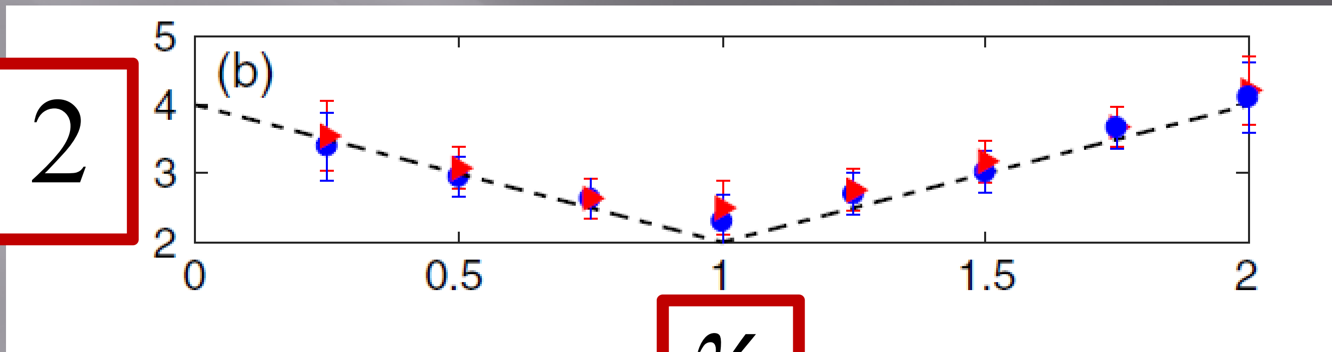


Duality of μ(γ) =μ(2−γ)

$$\exp[\langle \ln|\Psi|^2 \rangle] = \frac{c}{|i-i_0|^{\mu}}$$

$$V_{nm} = \frac{1}{|n-m|^{\gamma}}$$

$\mu > 2$



γ

All states are localized. More long-range hopping leads to stronger localization for γ<1

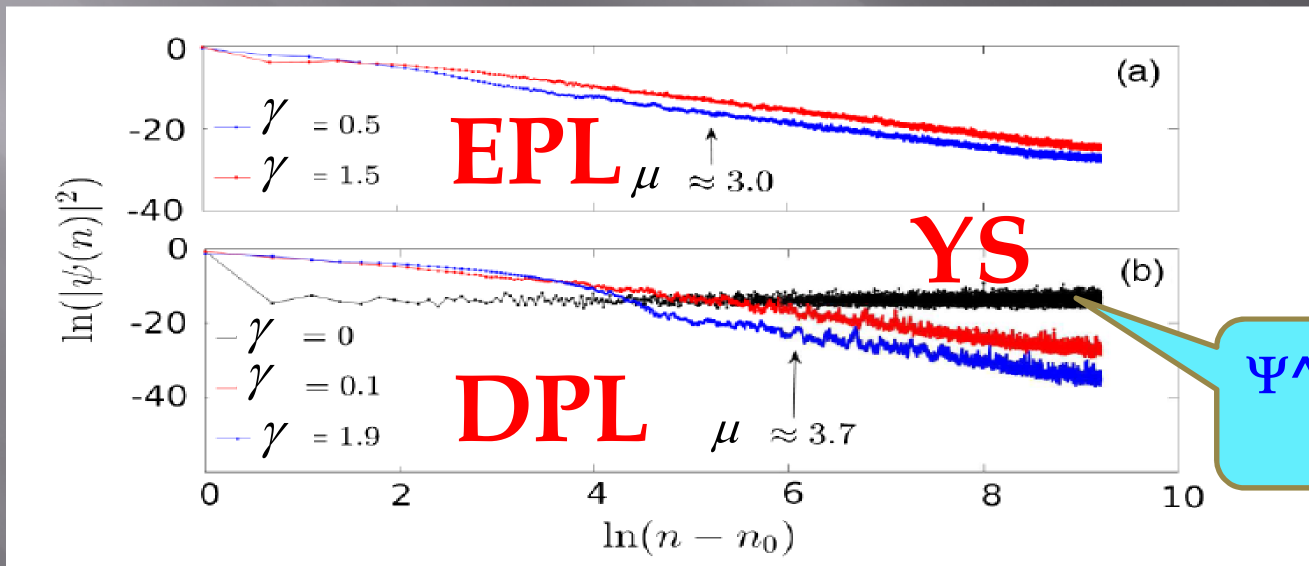
Yuzbashyan-Shastry model

$$H = \sum_{n} \varepsilon_n \; c_n^+ c_n - \sum_{n,m} g_n g_m \; c_n^+ c_m$$

$$\Psi_E(i) = c\,\frac{g_i}{E-\varepsilon_i}$$

$$g_n = \frac{1}{N^{a/2}}$$

$$\sum_{i=0}^{N-1} \frac{g_i^2}{E-\varepsilon_i} = -1$$



Ψ^{2}~1/N^{2}
if a<2

In DPL, EPL and YS models with long-range ($0<\gamma<1$), sign-definite hopping all states (except few at the lowest energies) are localized

What matters? Sign-preserving?

No, the same behavior in YS model with g_{n} drawn from Gaussian ensemble with <g_{n}>=0.

Correlations and not sign-preserving is the cause of localization at long-range hopping

$$V_{nm} = \frac{1+\eta_{nm}}{|n-m|^{\gamma}}$$

Even small noise η_nm kills localization at γ<1.

Outlook: exact solution for DPL model?

$$H = \sum_n \varepsilon_n \ c_n^+ c_n - \sum_{n,m} g_n g_m \ c_n^+ c_m$$

Degenerate kernel?

$$H = \sum_n \varepsilon_n \ c_n^+ c_n - \sum_{n,m} | n - m |^{-\gamma} \ c_n^+ c_m$$

Weakly singular (0<γ<1) kernel?

Conclusion

- Absence of delocalization in 1D systems with long-range hopping
- Duality in the exponent $\mu(\gamma)=\mu(2-\gamma)$ of power-law localization at long-range ($\gamma<1$) and short-range ($1<\gamma<2$) power-law hopping
- Exactly solvable models of 1D localization with correlated distance-independent hopping $V_{nm}=g_{n}g_{m}$
- Conjecture about exact solubility of 1D model with diagonal disorder and deterministic power-law hopping $V_{nm}=|n-m|^{-\gamma}$