Unconventional Magnons and their Impact on Spin Pumping Transport

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Outline/Results

- Introduction and motivation
- Quasiparticles in Ferromagnets with **spin $\hbar^* > \hbar$**
- Spin current shot noise and quantum of transport
- Quasiparticles in Ferrimagnets and Antiferromagnets
  → squeezing ($\hbar^* > \hbar$) and hybridization ($0 \leq \hbar^* < \hbar$)
- Spin pumping in Ferrimagnets and Antiferromagnets
  → interface asymmetries, cross sublattice coupling
- Phenomenology of non-standard Gilbert damping in Ferri- and Antiferromagnets

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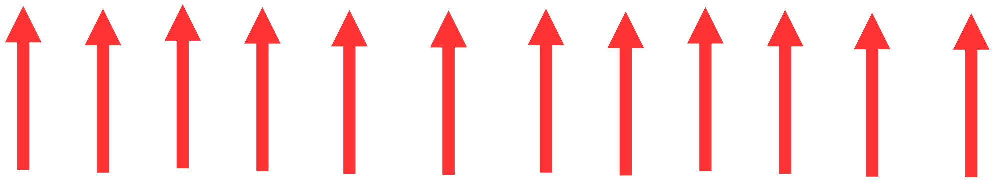
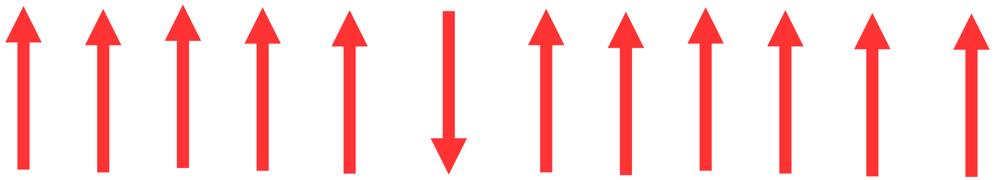
- Gilbert damping in Ferri- and Antiferromagnets

Quasiparticles in a ferromagnet



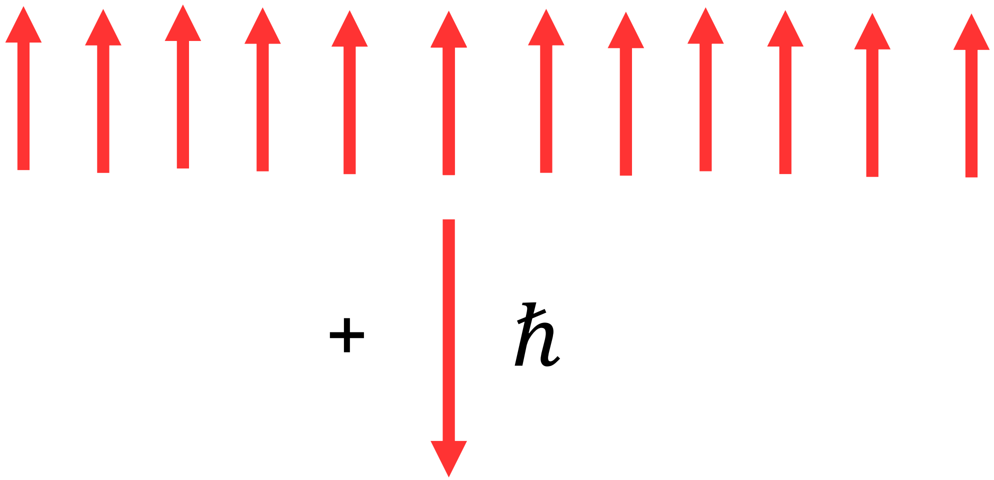
C. Kittel, *Introduction to Solid State Physics* (John Wiley & Sons, New York, 1953)

Quasiparticles in a ferromagnet



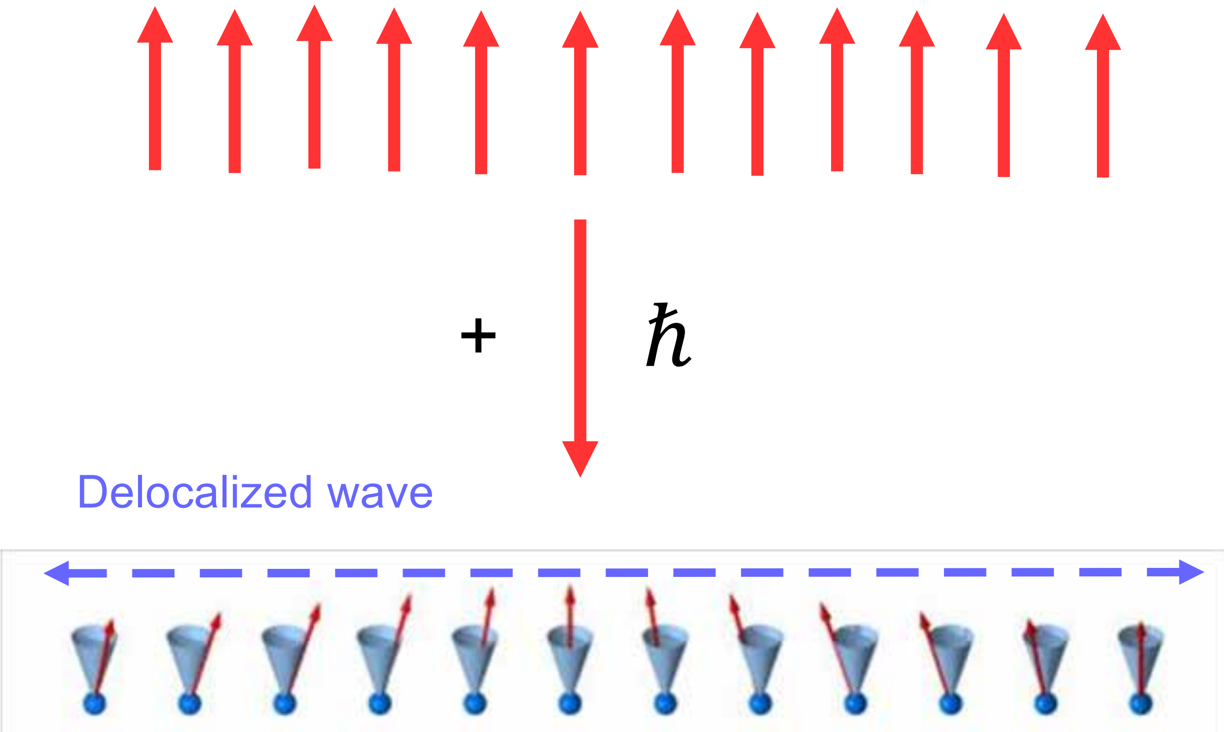
C. Kittel, *Introduction to Solid State Physics* (John Wiley & Sons, New York, 1953)

Magnon



C. Kittel, *Introduction to Solid State Physics* (John Wiley & Sons, New York, 1953)

Magnon



Considering only exchange interaction and Zeeman energy!

C. Kittel, *Introduction to Solid State Physics* (John Wiley & Sons, New York, 1953)

Quasiparticles in a ferromagnet

With magnon operators $\tilde{b}_{\vec{q}}$ and $\tilde{b}_{\vec{q}}^{+}$ ("bosons")

$$\tilde{\mathcal{H}}_{\mathrm{F}} = \sum_{\boldsymbol{q}} A_{\boldsymbol{q}} \tilde{b}_{\boldsymbol{q}}^{\dagger} \tilde{b}_{\boldsymbol{q}}$$

Effect of dipolar interactions!

Quasiparticles: squeezed magnons

Bogoliubov transformation
to new quasi-particles

$$\tilde{\beta}_{\boldsymbol{q}} = u_{\boldsymbol{q}} \tilde{b}_{\boldsymbol{q}} - v_{\boldsymbol{q}}^{*} \tilde{b}_{-\boldsymbol{q}}^{\dagger}$$

$$\tilde{\mathcal{H}}_{\mathrm{F}} = \sum_{\boldsymbol{q}} \hbar \omega_{\boldsymbol{q}} \tilde{\beta}_{\boldsymbol{q}}^{\dagger} \tilde{\beta}_{\boldsymbol{q}}$$

Ground state: squeezed vacuum

$$\tilde{\beta}_{\vec{q}} |\psi_G\rangle_{\vec{q}} = 0 \quad \rightarrow \quad |\psi_G\rangle_{\vec{q}} \neq |0\rangle$$

Classical Hamiltonian

$$\mathcal{H}_\mathrm{F} = \int_{V_\mathrm{F}} d^3r \, (H_\mathrm{Z} + H_\mathrm{aniso} + H_\mathrm{ex} + H_\mathrm{dip})$$

Linearization around equilibrium magnetization (M_s):
Magnetization saturated along z direction

$$H_Z + H_{aniso} = \frac{E_{za}}{M_s^2}\left(M_x^2 + M_y^2\right)$$

$$H_{ex} = \frac{A}{M_s^2}\left(\left(\vec{\nabla} M_x\right)^2 + \left(\vec{\nabla} M_y\right)^2\right)$$

C. Kittel, *Quantum Theory of Solids* (John Wiley & Sons, London 1963).

Classical Hamiltonian

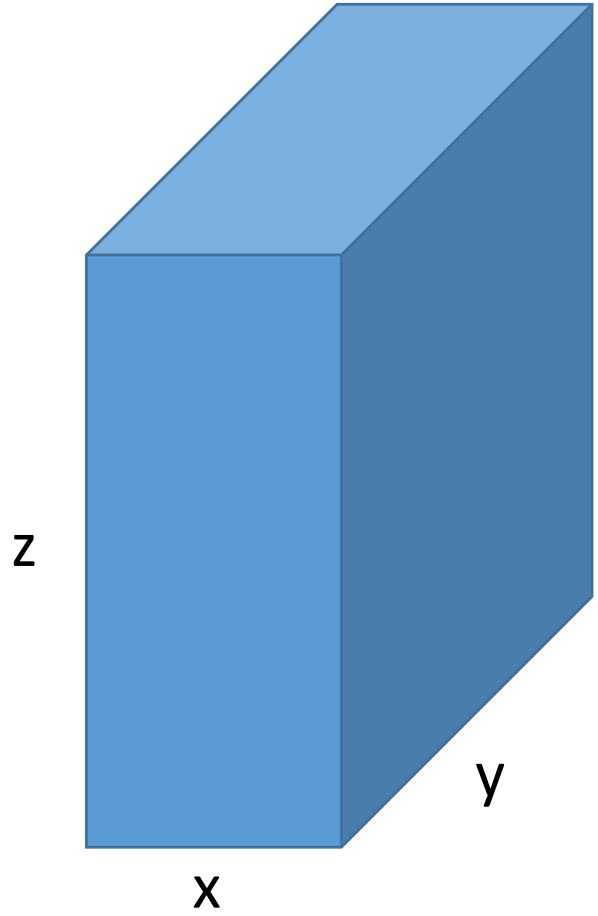
Simplification for dipolar interaction:

- Homogeneous demagnetization field
- Demagnetization tensor:

$$\hat{N} = \mathrm{diag}(N_x, N_y, N_z) = \mathrm{diag}(1,0,0)$$

$$H_{dip} = -\frac{1}{2}\mu_0 \vec{H}_m \vec{M} = \frac{1}{2}\mu_0 M_x^2$$

$$\vec{H}_m = -\hat{N}\vec{M} = -M_x \vec{e}_x$$



C. Kittel, *Quantum Theory of Solids* (John Wiley & Sons, London 1963).

Quantization: HP transformations

Magnon field operators in real space: $\tilde{b}(\vec{r})$, $\tilde{b}^+(\vec{r})$

Ladder operators: $\widetilde{M}_\pm = \widetilde{M}_x \pm i\widetilde{M}_y$ ($[\widetilde{M}_x, \widetilde{M}_y] = i\hbar\widetilde{M}_z$)

$$\widetilde{M}_+ = M_s\sqrt{1-\tilde{b}^+\tilde{b}/S}\,\tilde{b}$$

$$\widetilde{M}_- = M_s\tilde{b}^+\sqrt{1-\tilde{b}^+\tilde{b}/S}$$

$$\widetilde{M}_z = M_s - \hbar\tilde{b}^+\tilde{b}/S$$

$\approx$

$$\widetilde{M}_+ = M_s\,\tilde{b}$$

$$\widetilde{M}_- = M_s\,\tilde{b}^+$$

$$\widetilde{M}_z = M_s - \hbar\tilde{b}^+\tilde{b}$$

$$S = 2M_S/\gamma\hbar$$

$$\rightarrow \widetilde{M}_x^2 \sim \left(\tilde{b}_0^+ + \tilde{b}_0\right)^2 = 2\tilde{b}_0^+\tilde{b}_0 + \tilde{b}_0^+\tilde{b}_0^+ + \tilde{b}_0\tilde{b}_0$$

T. Holstein and H. Primakoff (HP), *Field Dependence of the Intrinsic Domain Magnetization of a Ferromagnet*, Phys. Rev. 58, 1098 (1940).

C. Kittel, *Quantum Theory of Solids* (John Wiley & Sons, London 1963).

Quantum Hamiltonian

Magnon annihilation operators in momentum space: $\tilde{b}_{\vec{q}}$

$$\tilde{H}_F = \tilde{H}_0 + \sum_{\vec{q}\neq 0} \tilde{h}_{\vec{q}} \quad \text{with} \quad \tilde{H}_0 = A_0 \tilde{b}_0^+ \tilde{b}_0 + \underbrace{B_0 \tilde{b}_0^+ \tilde{b}_0^+ + B_0 \tilde{b}_0 \tilde{b}_0}$$

$$A_0 = \hbar\omega_{za} + \hbar\omega_s/2$$
$$B_0 = \hbar\omega_s/4$$

→ Squeezing Hamitonian

$\hbar\omega_{za}$ = Anisotropy and external fields

$\hbar\omega_s = \mu_0 \mu_B M_s$ → Energy of a moment in the dipole field!

- Finite momentum Hamiltonian has similar form with $A_{\vec{q}}, B_{\vec{q}}$
- Hamiltonian can be diagonalized by a Bogolubov transformation (for each $(\vec{q}, -\vec{q})$ seperately)
- Non-trivial ground state and excitations

  → squeezed vacuum and squeezed magnons

Squeezed-magnons

Bogoliubov transformation to new quasi-particles

$$\tilde{\beta}_{\boldsymbol{q}} = u_{\boldsymbol{q}}\tilde{b}_{\boldsymbol{q}} - v^*_{\boldsymbol{q}}\tilde{b}^\dagger_{-\boldsymbol{q}}$$

$$\tilde{\mathcal{H}}_{\mathrm{F}} = \sum_{\boldsymbol{q}} \hbar\omega_{\boldsymbol{q}}\tilde{\beta}^\dagger_{\boldsymbol{q}}\tilde{\beta}_{\boldsymbol{q}}$$

E.g. for $\vec{q} = 0$

- $\omega_0 = \sqrt{\omega_{za}^2 + \omega_{za}\omega_s}$
- $u_0^2 - v_0^2 = 1$
- $v_0^2 = \frac{\omega_s^2/4\omega_0}{\omega_0+\omega_{za}+\omega_s/2}$

Squeezed Vacuum

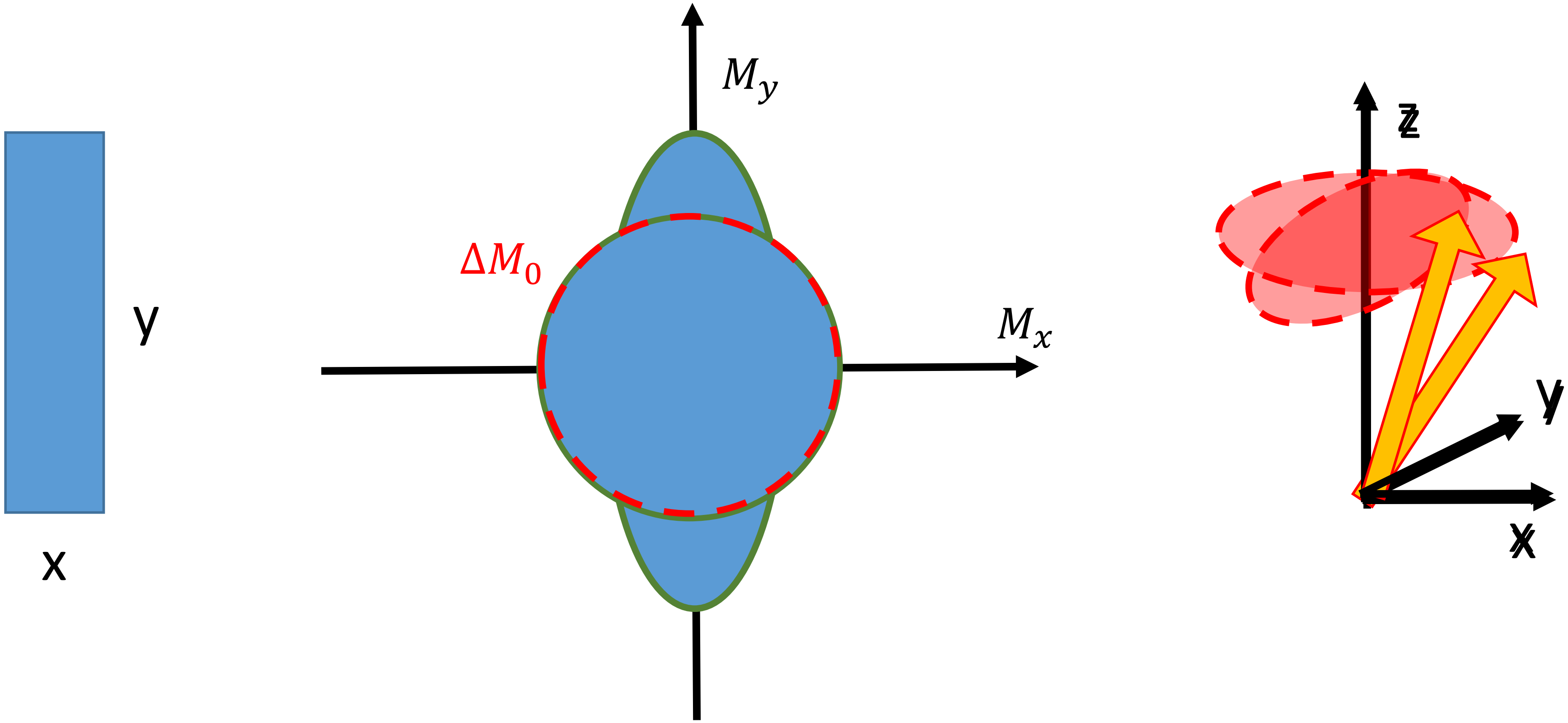
Ground state defined by

$$\tilde{\beta}_{\vec{q}}|\psi_G\rangle_{\vec{q}} = 0 \quad \leftrightarrow \quad |\psi_G\rangle_{\vec{q}} \sim |0\rangle_{\vec{q}} + \frac{v_{\vec{q}}}{u_{\vec{q}}}|2\rangle_{\vec{q}} + \cdots$$

Ground state: Squeezed Vacuum

Heisenberg uncertainty, viz. $\langle \Delta\widetilde{M}_x^2 \rangle \langle \Delta\widetilde{M}_y^2 \rangle \geq \Delta M_0^4$

$$\langle \Delta\widetilde{M}_x^2 \rangle = \Delta M_0^2 e^{-\xi} \quad \langle \Delta\widetilde{M}_y^2 \rangle = \Delta M_0^2 e^{+\xi} \quad \tanh \xi = \frac{v_0}{u_0}$$

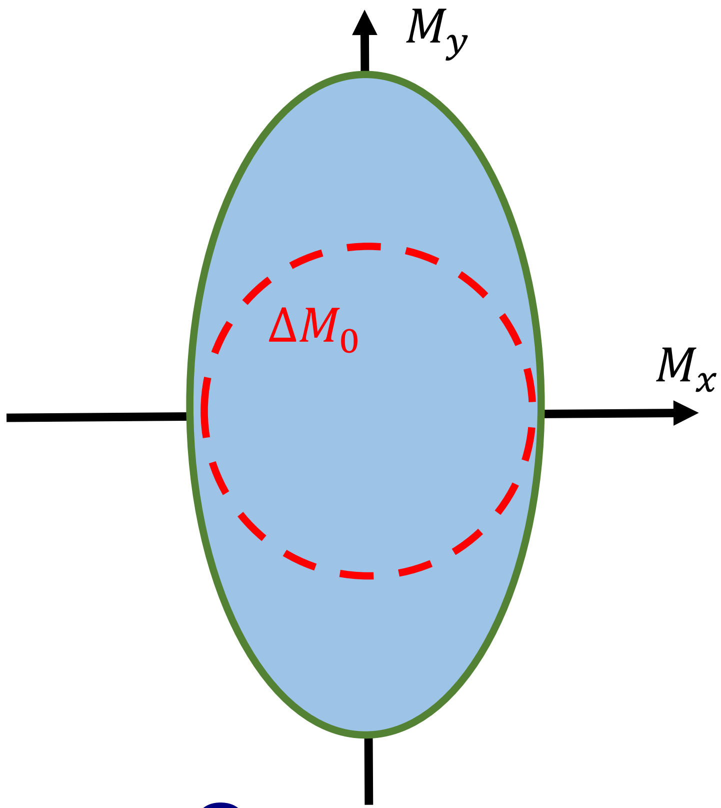


A Kamra and W. Belzig, Super-Poissonian shot noise of squeezed-magnon mediated spin transport, Phys. Rev. Lett. 116, 146601 (2016).

Squeezed magnons

$$
\begin{aligned}
|n_\beta = 1\rangle &= \tilde{\beta}_0^+ |\psi_G\rangle \\
&= (u_0 \tilde{b}^+ - v_0 \tilde{b}_0)|\psi_G\rangle \\
&\sim |1\rangle + \cdots |3\rangle + \cdots |5\rangle + \cdots
\end{aligned}
$$

→ Complex superposition of elementary magnons



Spin of squeezed magnon?

$$\langle n_\beta = 1|\tilde{S}_z|n_\beta = 1\rangle - \langle\psi_G|\tilde{S}_z|\psi_G\rangle = \hbar(1 + 2v_0^2) \equiv \hbar^* \geq \hbar$$

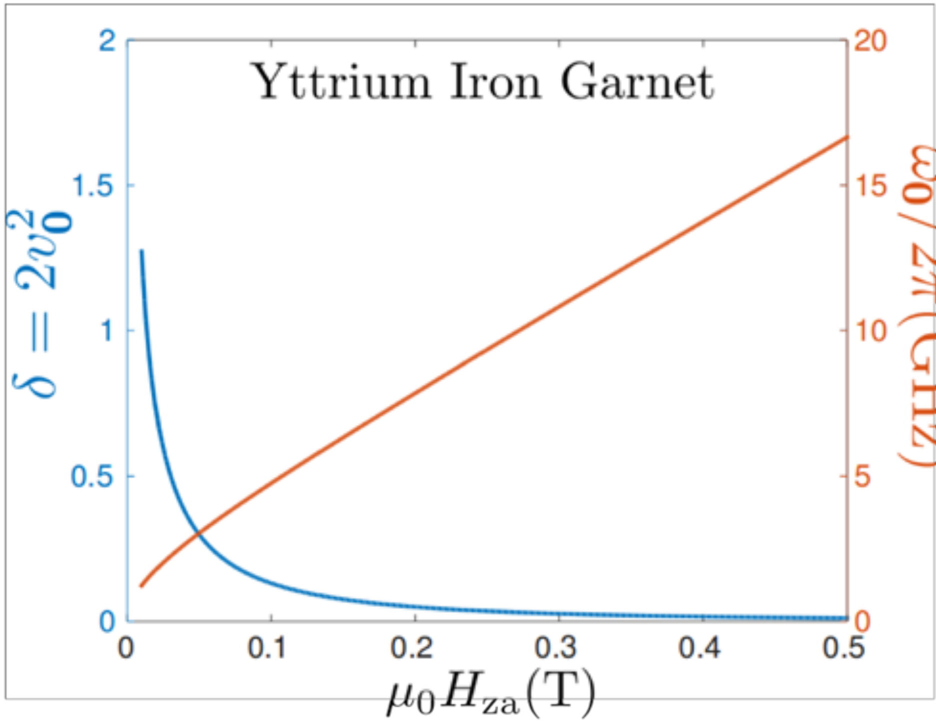
Note: Bogolubov trafo depends in general on $\vec{q}$: $\hbar^* \rightarrow \hbar^*_{\vec{q}}$

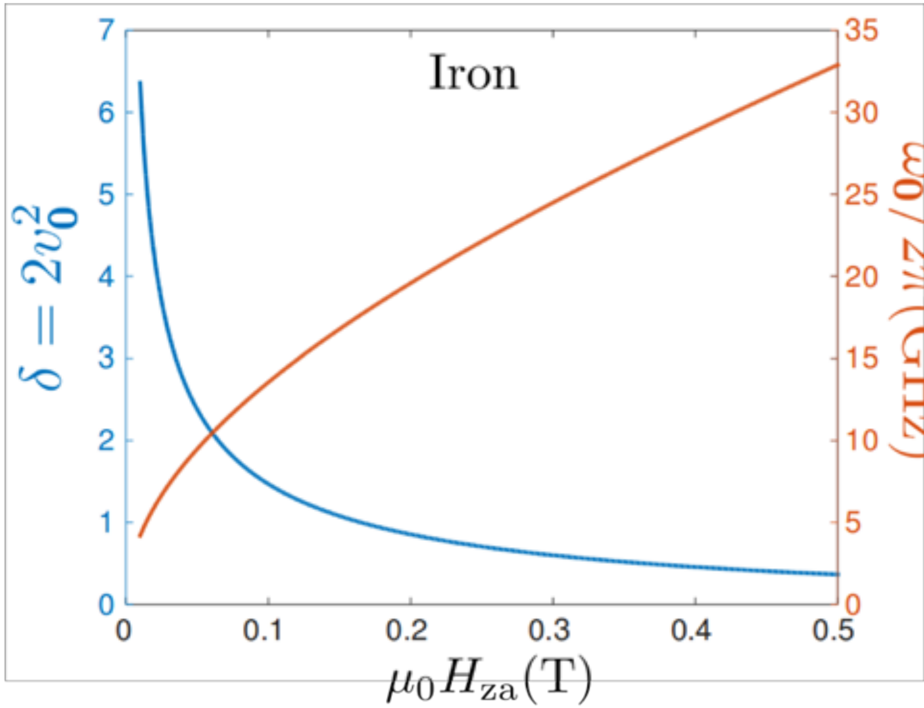
A Kamra and W. Belzig, Super-Poissonian shot noise of squeezed-magnon mediated spin transport, Phys. Rev. Lett. 116, 146601 (2016).

Spin of squeezed magnon

$$\hbar^* = \hbar(1 + 2v_0^2)$$

$$2v_0^2 = \frac{\omega_s^2/4\omega_0}{\omega_0 + \omega_{za} + \omega_s/2} \qquad \omega_0 = \sqrt{\omega_{za}^2 + \omega_{za}\omega_s} \qquad \omega_{za} = \mu_0 \gamma H_{za}$$





→ Large $\hbar^*$ for realistic material parameters

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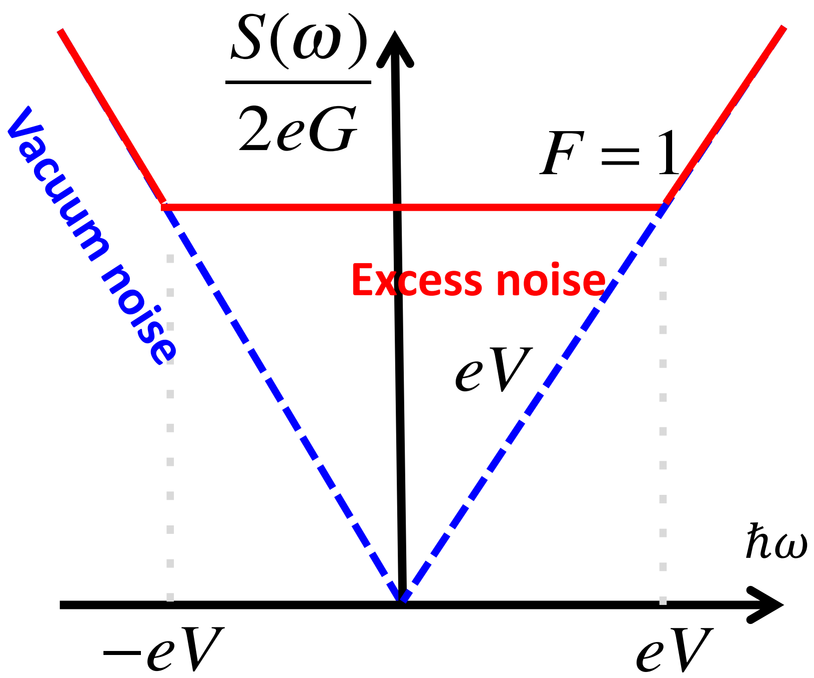
Quantum noise spectral density of a quantum point contact

Conductance $G = G_Q \sum_n T_n$ Conductance quantum $G_Q = \frac{2e^2}{h}$

Fano factor $F = \sum_n T_n (1 - T_n) / \sum_n T_n$ Transmission probabilities T_n

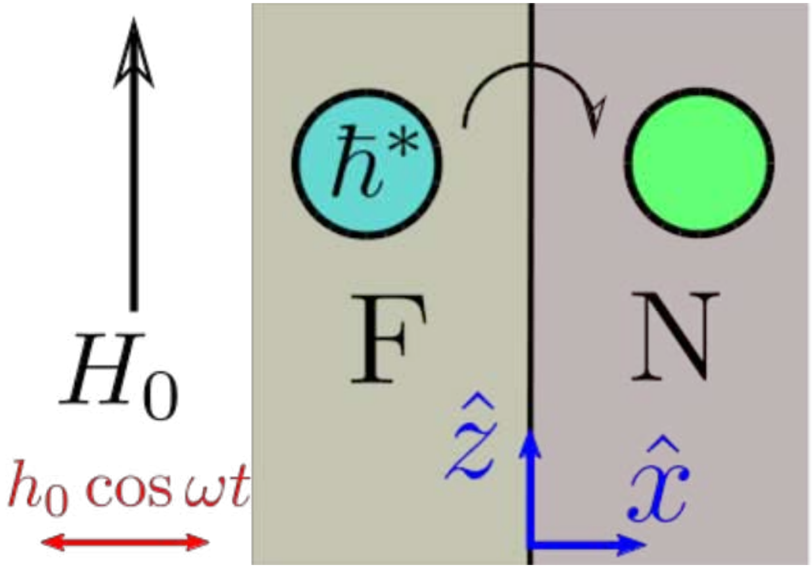
$$S(\omega) = \int dt e^{i\omega t} \left\langle \left\{ \hat{I}(0), \hat{I}(t) \right\} \right\rangle = 2eG \left[F \left(eV - |\omega| \right) \theta(eV - |\omega|) + |\omega| \right]$$



- Quantum noise spectral density is symmetric and has kinks for ω=eV
- Reveals the time scale of the junction ħω=eV (for a perfectly linear current-voltage characteristic!)

E.g. Lesovik (1997), Blanter and Büttiker Phys. Rep. (2000)

Spin detection via spin pumping



$$\hbar^* = \hbar(1 + 2|v_{\mathbf{0}}|^2)$$

„Tunneling“ Hamitonian

$$\widetilde{H}_{int} = \int d^2\rho J_I \widetilde{\vec{M}}(\vec{\rho})\tilde{\vec{s}}_N(\vec{\rho})$$

$$= \sum_{\vec{k}\vec{k}'\vec{q}} J_{\vec{k}\vec{k}'\vec{q}} \tilde{b}_{\vec{q}} \tilde{c}^{+}_{\vec{k}\uparrow} \tilde{c}_{\vec{k}'\downarrow} + h.c.$$

2nd–order perturbation in J for the z-component of the spin current I_{dc}:

$$I_{dc} = G_s \hbar^* \omega |\chi|^2$$

Spin conductance
$G_s = \pi J_I^2 \nu_N^2$

FMR-induced amplitude

A Kamra and W. Belzig, Super-Poissonian shot noise of squeezed-magnon mediated spin transport, Phys. Rev. Lett. 116, 146601 (2016).

Spin detection via **spin current shot noise**

Spin noise: uncorrelated transfer of bunches $\hbar^*$(Poissonian statistics) in time period t_0

Statistics of number of events N:

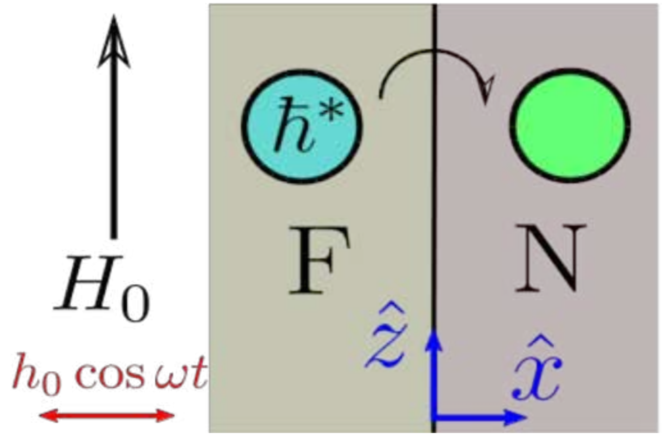
$\langle N \rangle = \bar{N} = \Gamma t_0$

$\langle \Delta N^2 \rangle = \bar{N} = \Gamma t_0$

Statistics of Spin $S = \hbar^* N$:

$\langle S \rangle = \hbar^* \bar{N} = \hbar^* \Gamma t_0$

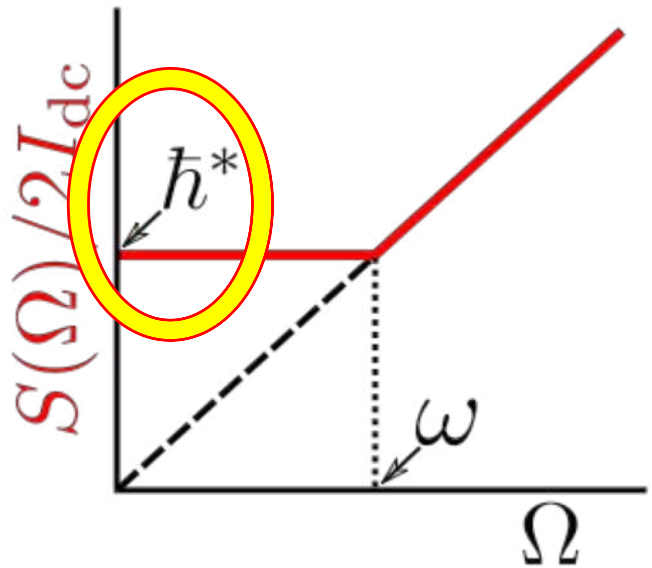
$\langle \Delta S^2 \rangle = {\hbar^*}^2 \langle \Delta N^2 \rangle = \hbar^* \langle S \rangle$



$\hbar^* = \hbar(1 + 2v_0^2)$

Full noise spectral density

$$S(\Omega) = 2 \int dt e^{i\Omega t} \langle \Delta I_z(t) \Delta I_z \rangle$$
$$= {\hbar^*}^2 G_s |\chi|^2 [|\omega - \Omega| + |\omega + \Omega|]$$



- Zero-frequency noise $S(0) = 2\hbar^* I_{dc}$
- Analog electric current noise with $\omega = eV$!

A Kamra and W. Belzig, Super-Poissonian shot noise of squeezed-magnon mediated spin transport, Phys. Rev. Lett. 116, 146601 (2016).

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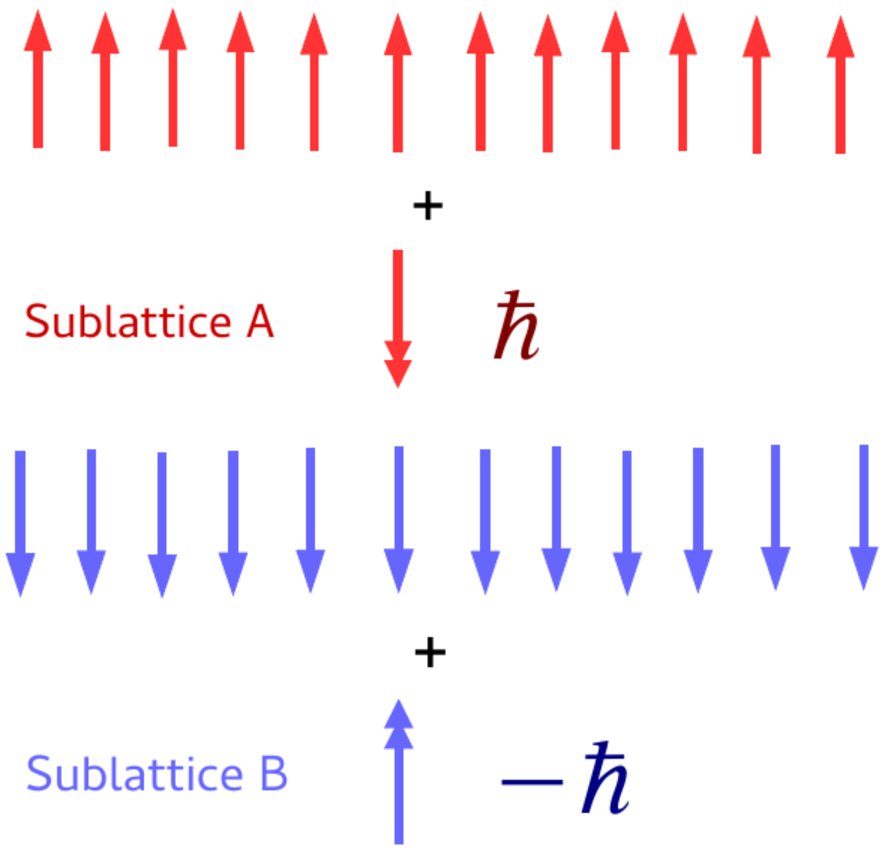
Two interpenetrating sublattices



Similar model as before, but generalized to two sublattices:

$$J \to J_{AA}, J_{BB}, J_{AB} \quad E_{za} \to E_{za}^{A/B} \quad M_s \to M_{A0/B0} \quad \tilde{b}_{\vec{q}} \to \tilde{a}_{\vec{q}}, \tilde{b}_{\vec{q}}$$

Two-sublattice magnon Hamiltonian

$$\tilde{\mathcal{H}} = \sum_{\boldsymbol{k}} \left[\frac{A_{\boldsymbol{k}}}{2} \tilde{a}_{\boldsymbol{k}}^{\dagger} \tilde{a}_{\boldsymbol{k}} + \frac{B_{\boldsymbol{k}}}{2} \tilde{b}_{\boldsymbol{k}}^{\dagger} \tilde{b}_{\boldsymbol{k}} + C_{\boldsymbol{k}} \tilde{a}_{\boldsymbol{k}} \tilde{b}_{-\boldsymbol{k}} + D_{\boldsymbol{k}} \tilde{a}_{\boldsymbol{k}} \tilde{a}_{-\boldsymbol{k}} + E_{\boldsymbol{k}} \tilde{b}_{\boldsymbol{k}} \tilde{b}_{-\boldsymbol{k}} + F_{\boldsymbol{k}} \tilde{a}_{\boldsymbol{k}} \tilde{b}_{\boldsymbol{k}}^{\dagger} \right] + \text{H.c.}$$

A_k, B_k : **intra**-sublattice exchange and external field
C_k : **inter**-sublattice exchange
D_k, E_k : dipolar interaction-induced **squeezing**
F_k : dipolar interaction-induced **hybridization**

4-D Bogoliubov Transform

Ferromagnet: $\tilde{b}_q \rightarrow \tilde{\beta}_q$

$$\tilde{\beta}_{\boldsymbol{q}} = u_{\boldsymbol{q}} \tilde{b}_{\boldsymbol{q}} - \underbrace{v^{*}_{\boldsymbol{q}} \tilde{b}^{\dagger}_{-\boldsymbol{q}}}_{\text{Dipolar interaction!}}$$

Ferrimagnet: $\tilde{a}_q, \tilde{b}_q \rightarrow \tilde{\alpha}_q, \tilde{\beta}_q$

$$\tilde{\alpha}_{\boldsymbol{q}} = u_{\boldsymbol{q}} \tilde{a}_{\boldsymbol{q}} + v_{\boldsymbol{q}} \tilde{b}^{\dagger}_{-\boldsymbol{q}} + \underbrace{w_{\boldsymbol{q}} \tilde{a}^{\dagger}_{-\boldsymbol{q}} + x_{\boldsymbol{q}} \tilde{b}_{\boldsymbol{q}}}_{\text{Dipolar interaction!}}$$

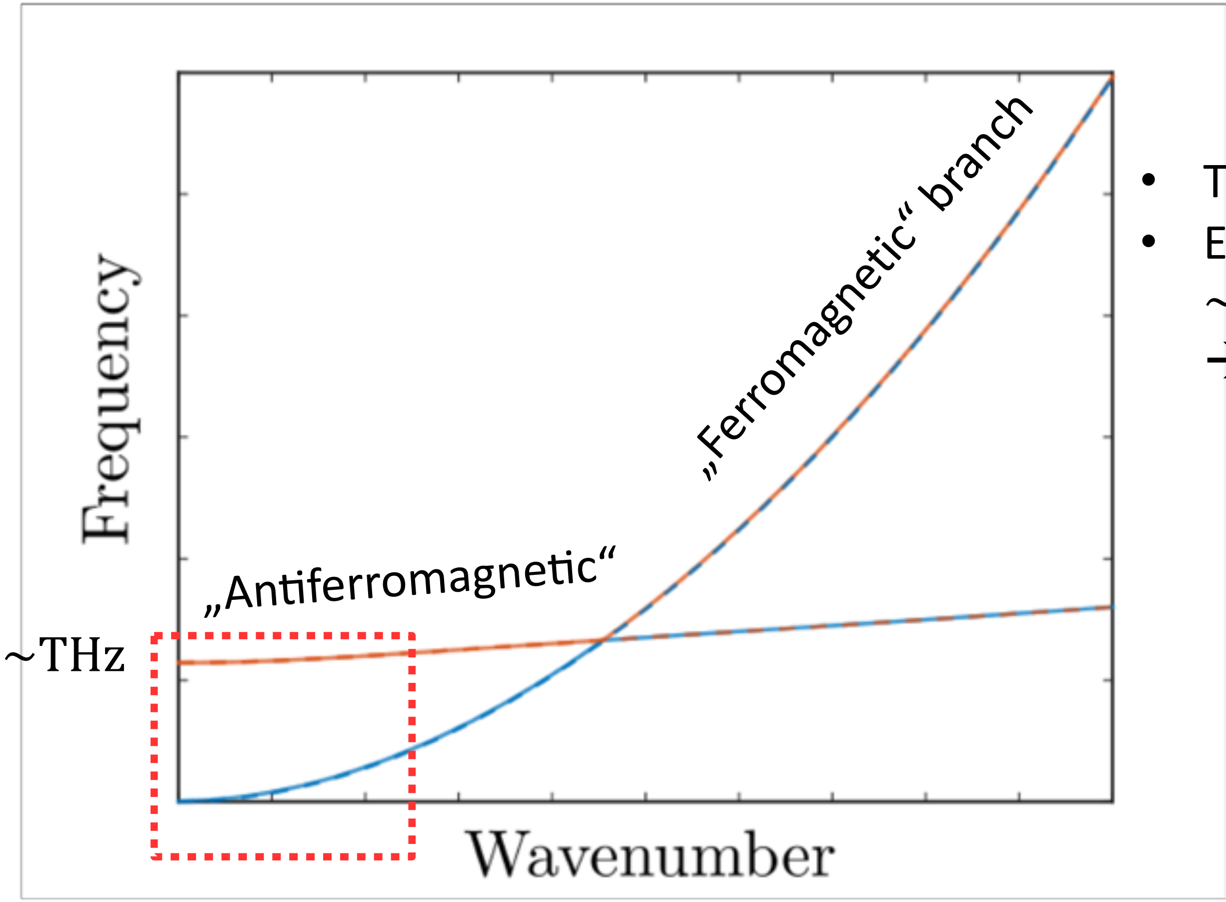
Magnon spin: $\hbar^{*} = \hbar \left(1 + 2|w_{\vec{q}}|^2 - 2|x_{\vec{q}}|^2\right)$

Squeezing → $\hbar^{*} > 1$

Hybridization → $\hbar^{*} < 1$

A. Kamra, U. Agrawal, and W Belzig, Noninteger-spin magnonic excitations in untextured magnets, Phys. Rev. B, 96, 020411(R) (2017).

Magnons in Ferrimagnets



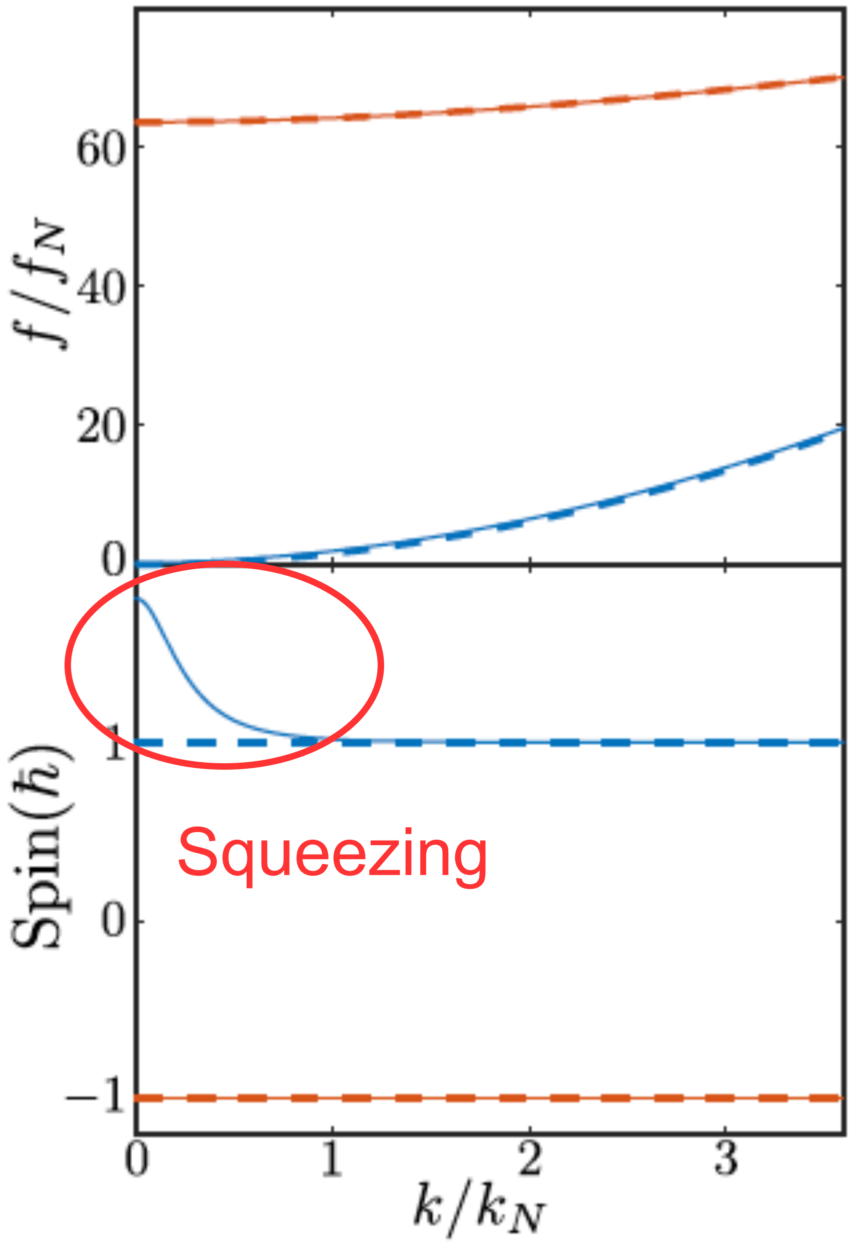
- Two branches crossing
- Effect of dipolar interaction ~GHz (solid lines) → anticrossing

A. Kamra, U. Agrawal, and W Belzig, Noninteger-spin magnonic excitations in untextured magnets, Phys. Rev. B, 96, 020411(R) (2017).

Ferrimagnet

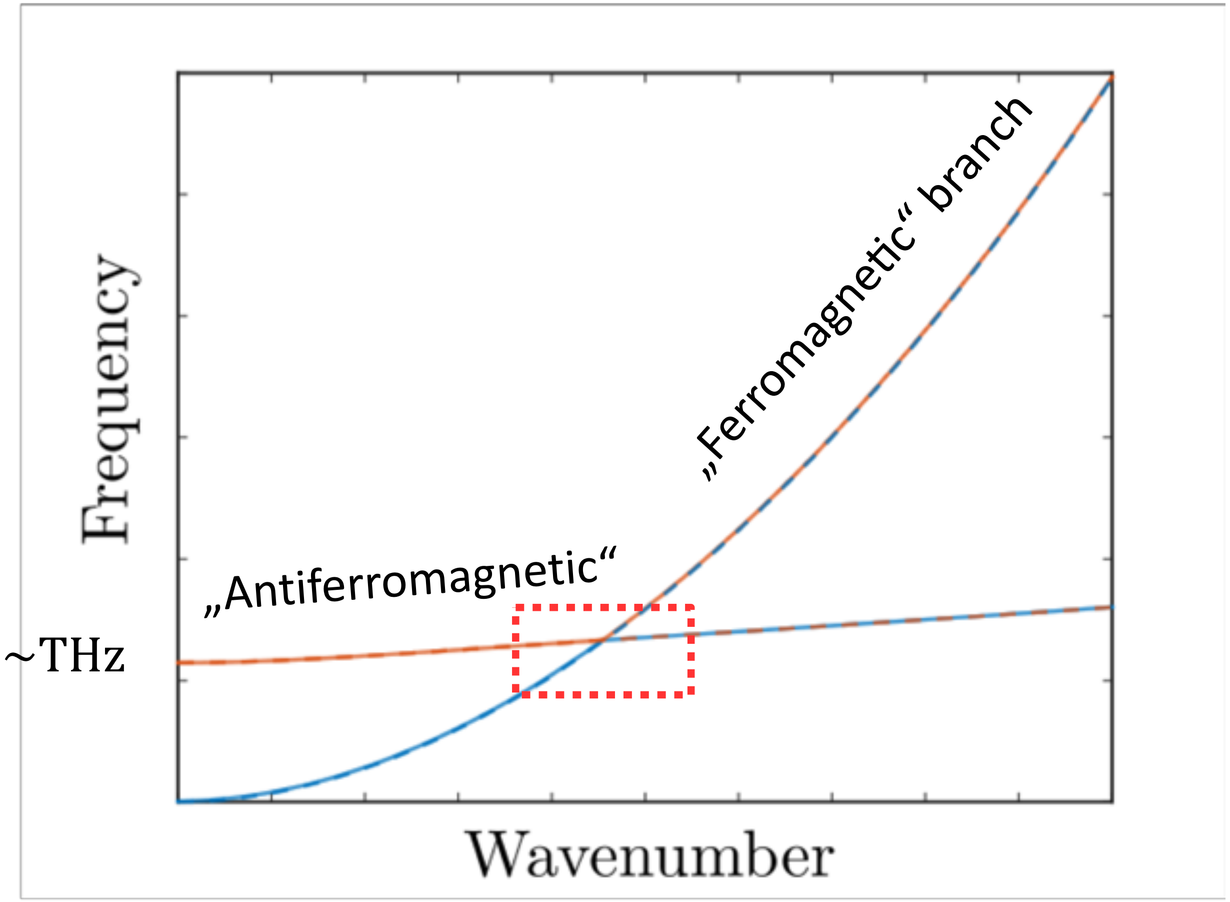
$M_{A0} = 5M_{B0}$

„squeezing“
(analog quasi-ferromagnet)



A. Kamra, U. Agrawal, and W Belzig, Noninteger-spin magnonic excitations in untextured magnets, Phys. Rev. B, 96, 020411(R) (2017).

Ferrimagnets

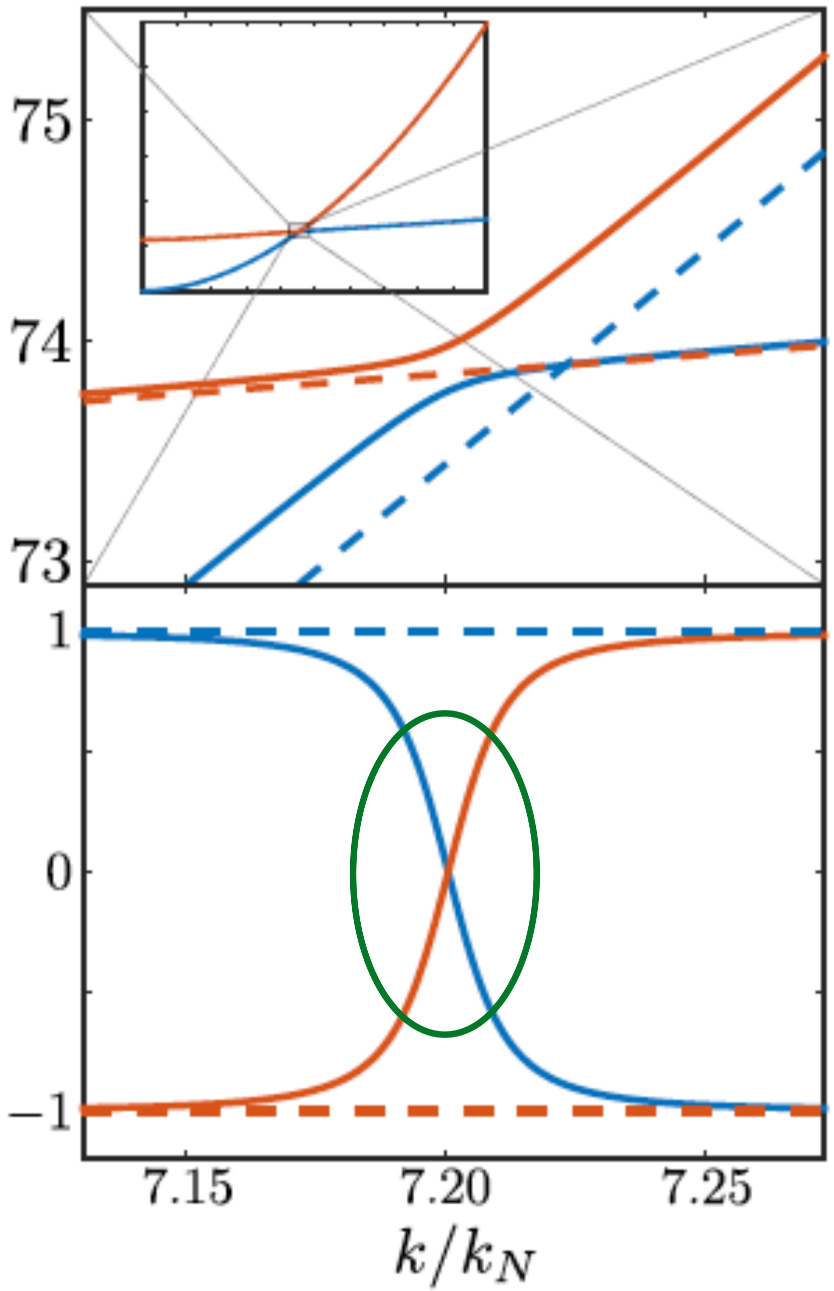


A. Kamra, U. Agrawal, and W Belzig, Noninteger-spin magnonic excitations in untextured magnets, Phys. Rev. B, 96, 020411(R) (2017).

Ferrimagnet

$M_{A0} = 2M_{B0}$

Hybridization



A. Kamra, U. Agrawal, and W Belzig, Noninteger-spin magnonic excitations in untextured magnets, Phys. Rev. B, 96, 020411(R) (2017).

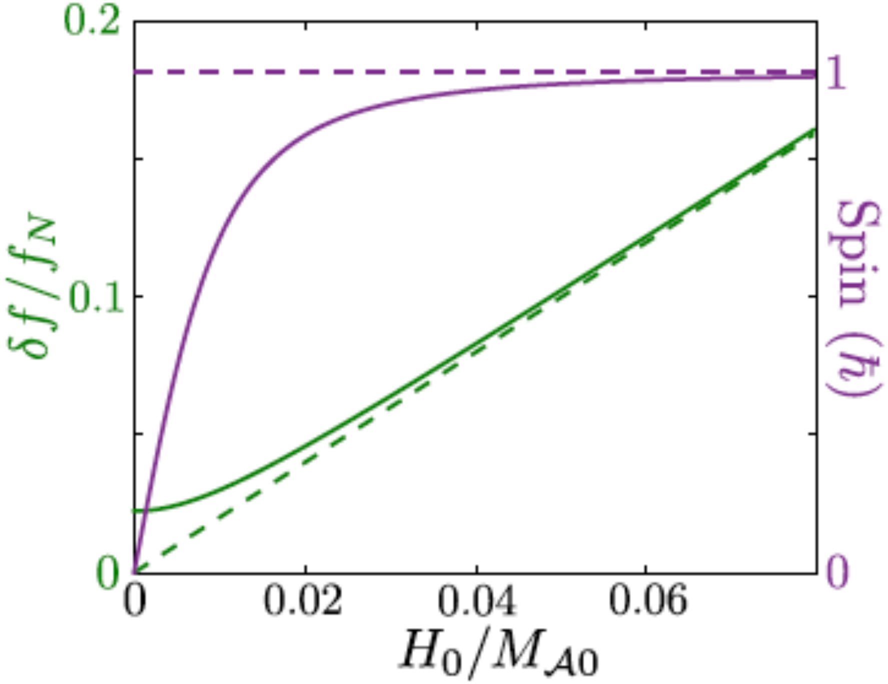
Antiferromagnets

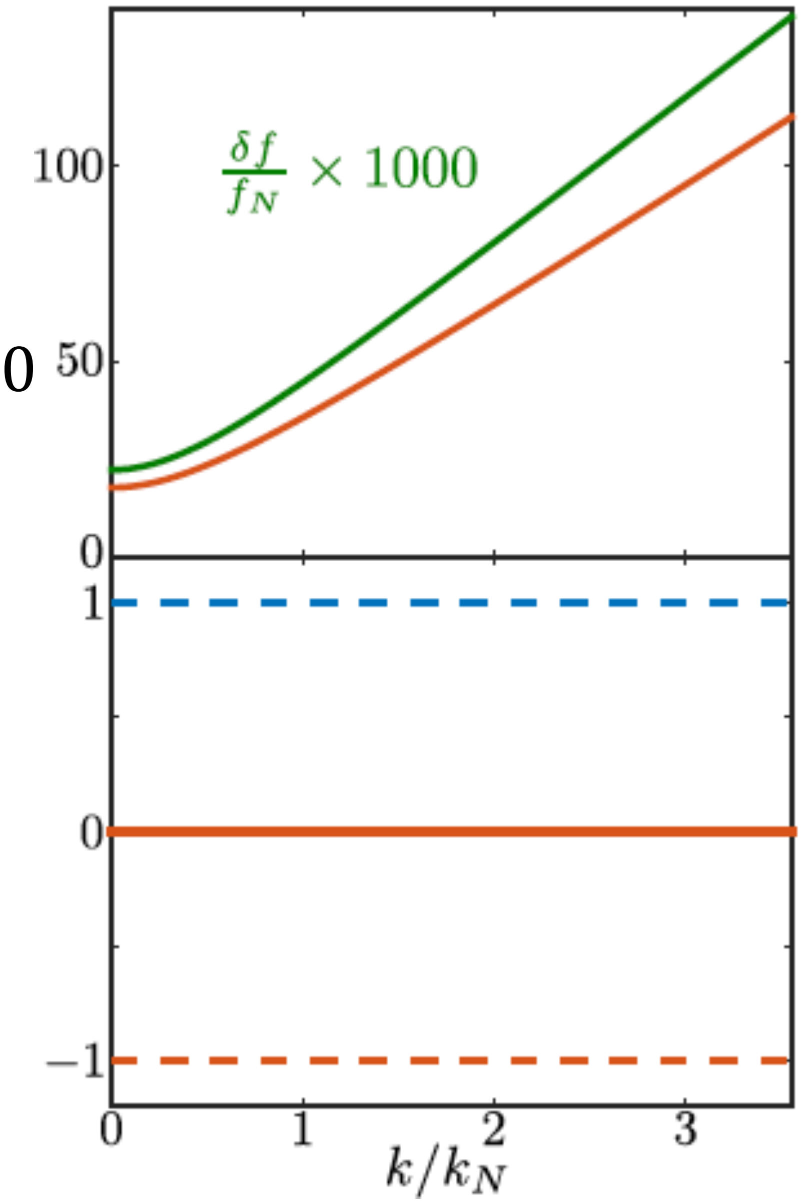
$$M_{A0} = 2M_{B0}$$

Magnon spin:

$$\hbar^* = \hbar\left(1 + 2|w_{\vec{q}}|^2 - 2|x_{\vec{q}}|^2\right) = 0$$

Zero-spin quasiparticles!

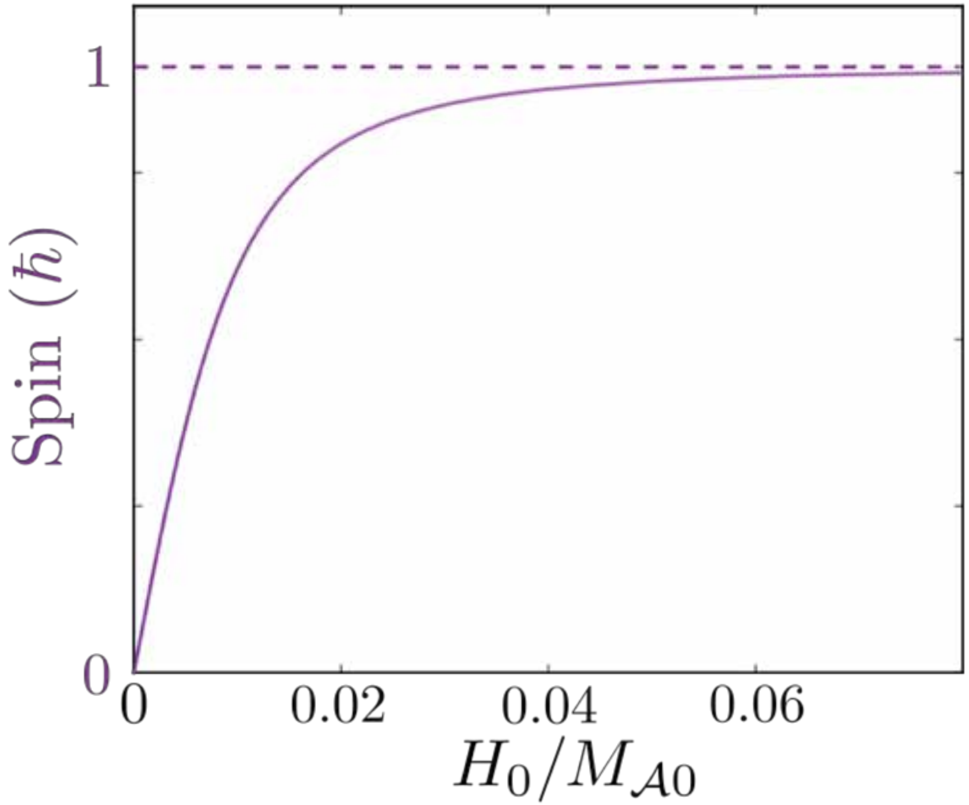




A. Kamra, U. Agrawal, and W Belzig, Noninteger-spin magnonic excitations in untextured magnets, Phys. Rev. B, 96, 020411(R) (2017).

Antiferromagnets

Zero-spin quasiparticles!



A. Kamra, U. Agrawal, and W Belzig, Noninteger-spin magnonic excitations in untextured magnets, Phys. Rev. B, 96, 020411(R) (2017).

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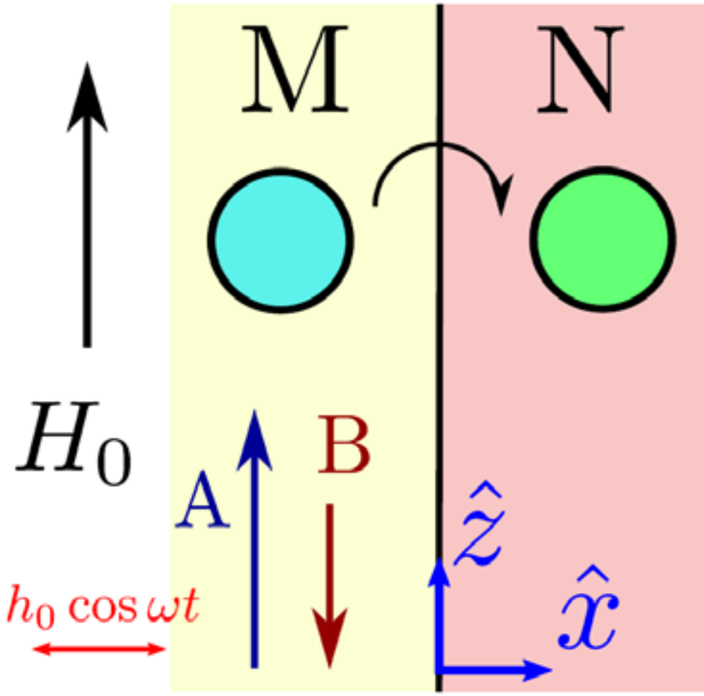
Spin pumping in ferrimagnets

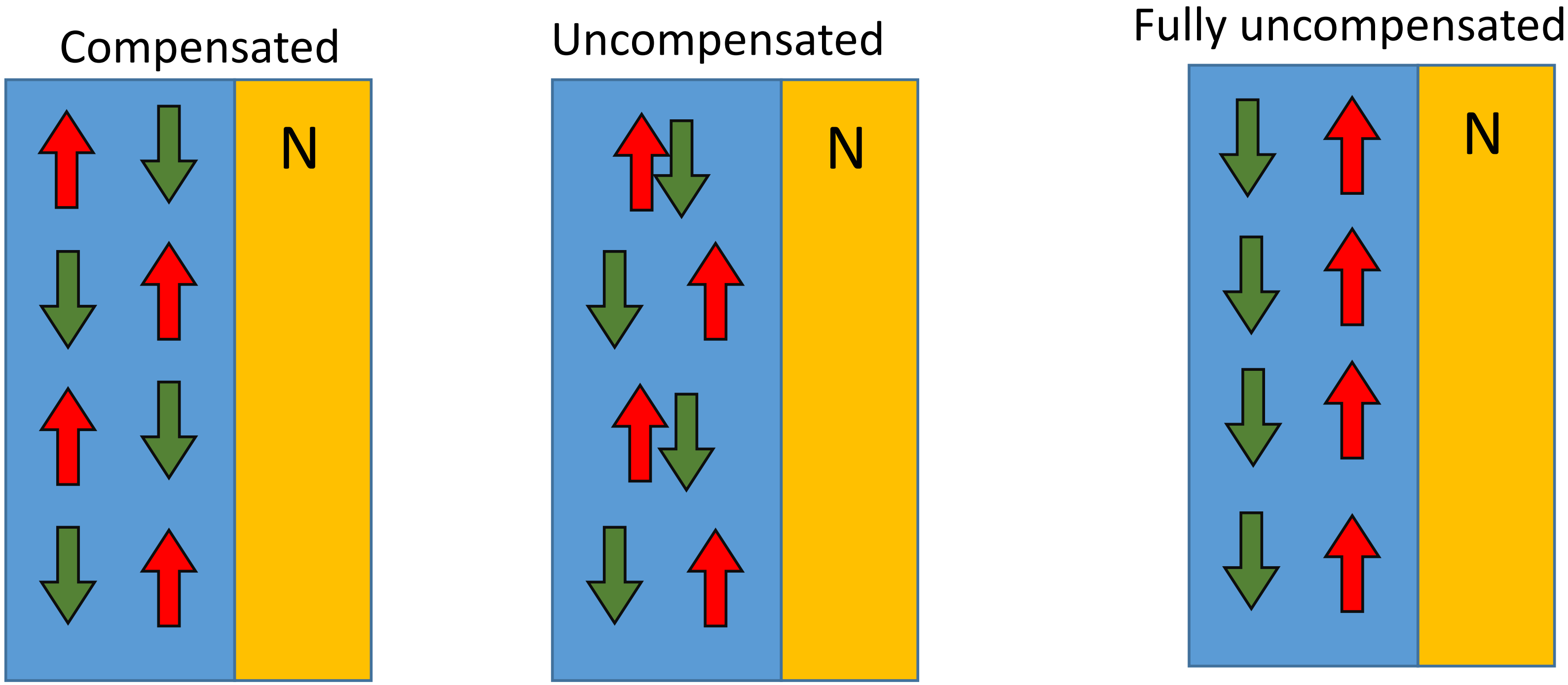


Bulk and Interfacial asymmetry!

A. Kamra and W Belzig, Phys. Rev. Lett. 119, 197201 (2017)

Spin pumping and shot noise in ferrimagnets: bridging ferro- and antiferromagnets

Interface (matters!)



A/B sublattice

$$\widetilde{H}_{int} = -\frac{1}{\hbar^2}\int d^2\rho\,[J_{iA}\widetilde{\vec{M}}_A(\vec{\rho}) + J_{iB}\widetilde{\vec{M}}_B(\vec{\rho})]\widetilde{\vec{s}_N}(\vec{\rho})$$

A. Kamra and W Belzig, Phys. Rev. Lett. 119, 197201 (2017)
Spin pumping and shot noise in ferrimagnets: bridging ferro- and antiferromagnets

Spin pumping in antiferromagnets (previous work)

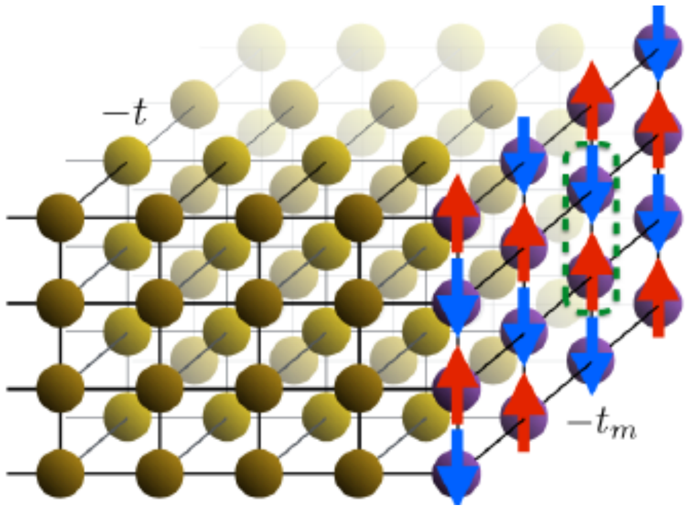
Different interface model and assumptions:

$$\frac{e}{\hbar}\boldsymbol{I}_s = G_r(\boldsymbol{n} \times \dot{\boldsymbol{n}} + \boldsymbol{m} \times \dot{\boldsymbol{m}})$$

G_r: real part of the mixing conductance

$$\vec{m} = \frac{1}{2}(\vec{m}_A + \vec{m}_B)$$

$$\vec{n} = \frac{1}{2}(\vec{m}_A - \vec{m}_B)$$



R. Cheng, J. Xiao, Q. Niu, and A. Brataas, Spin pumping and spin transfer torques in Antiferromagnets, Phys. Rev. Lett. 113, 057601 (2014).

Spin pumping in ferrimagnets

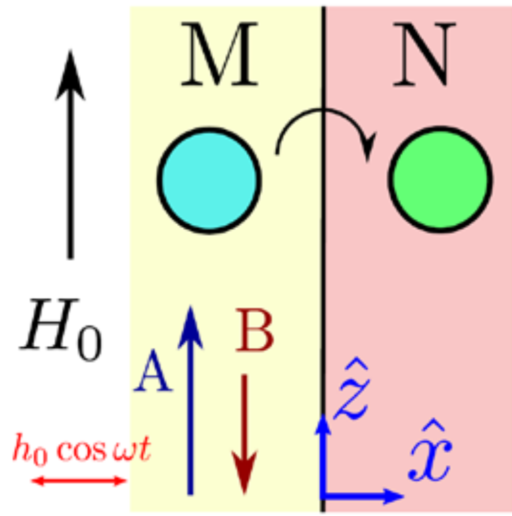
$$I_{sz} = 2\hbar|\chi|^2 \left[\Gamma_{AA}\left(|u|^2 - |w|^2\right) + \Gamma_{BB}\left(|v|^2 - |x|^2\right) - 2\Gamma_{AB}\Re\left(u^* v - w x^*\right)\right],$$

With $\Gamma_{AA} \sim J_{iA}^2$ $\Gamma_{BB} \sim J_{iB}^2$ $\Gamma_{AB} \sim J_{iA} J_{iB}$

In terms of sublattice magnetizations:

$$I_{sz} = G_{AA}\left(\vec{m}_A \times \dot{\vec{m}}_A\right)_z + G_{BB}\left(\vec{m}_B \times \dot{\vec{m}}_B\right)_z + G_{AB}\left(\vec{m}_A \times \dot{\vec{m}}_B + \vec{m}_B \times \dot{\vec{m}}_A\right)_z$$

$$G_{AA} \sim J_A^2 M_A^2 \nu_N^2, \ldots$$



- Under generic assumptions: $G_{AA} G_{BB} = G_{AB}^2$
- In general NO direct connection to $\hbar^*$
- Shot noise → detailed information on quasiparticles (provided $G_{AA/BB}$ are known)

A. Kamra and W Belzig, Phys. Rev. Lett. 119, 197201 (2017)

Spin pumping in ferrimagnets

$$\frac{e}{\hbar} I_{sz} = G_{AA}(\vec{m}_A \times \dot{\vec{m}}_A)_z + G_{BB}(\vec{m}_B \times \dot{\vec{m}}_B)_z + G_{AB}(\vec{m}_A \times \dot{\vec{m}}_B + \vec{m}_B \times \dot{\vec{m}}_A)_z$$

$$= G_{mm}(\vec{m} \times \dot{\vec{m}})_z + G_{nn}(\vec{n} \times \dot{\vec{n}})_z + G_{mn}(\vec{m} \times \dot{\vec{n}} + \vec{n} \times \dot{\vec{m}})_z$$

$$\vec{m} = \frac{1}{2}(\vec{m}_A + \vec{m}_B)$$

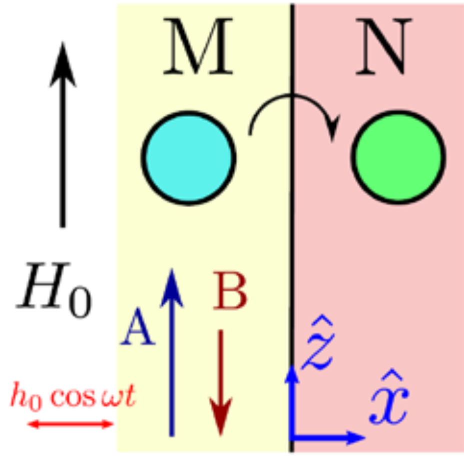
$$\vec{n} = \frac{1}{2}(\vec{m}_A - \vec{m}_B)$$

$$G_{mm} = G_{AA} + G_{BB} + 2G_{AB}$$

$$G_{nn} = G_{AA} + G_{BB} - 2G_{AB}$$

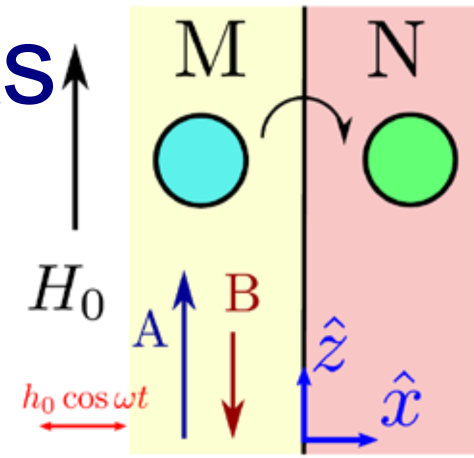
$$G_{mn} = G_{AA} - G_{BB}$$

$$G_{AB} = \sqrt{G_{AA} G_{BB}}$$



A. Kamra and W Belzig, Phys. Rev. Lett. 119, 197201 (2017)

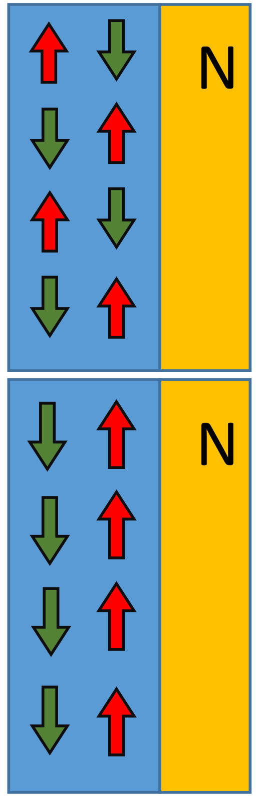
Spin pumping in ferrimagnets



$$G_{mm} = G_{AA} + G_{BB} + 2G_{AB}$$

$$G_{nn} = G_{AA} + G_{BB} - 2G_{AB} \qquad G_{AB} = \sqrt{G_{AA}G_{BB}}$$

$$G_{mn} = G_{AA} - G_{BB}$$



- Compensated interface:

$$G_{AA} = G_{BB} \rightarrow G_{mn} = 0 = G_{nn}$$

- Fully uncompensated interface

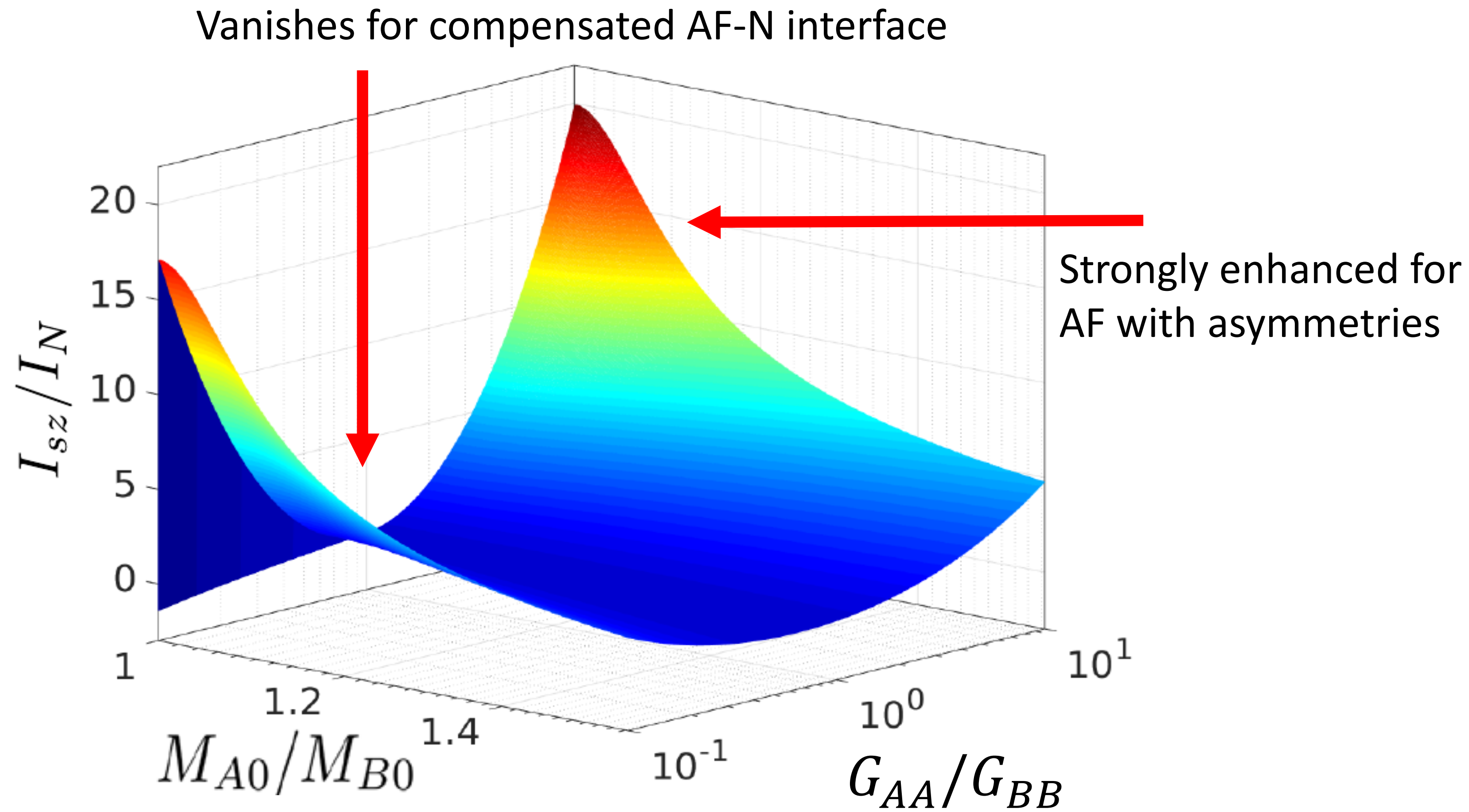
$$G_{BB} = 0 \rightarrow G_{mm} = G_{nn} = G_{mn}$$

- Chen et al.: $G_{AA} = G_{BB}\,, G_{AB} = 0$

$$\rightarrow \quad G_{mm} = G_{nn}\,, G_{mn} = 0$$

A. Kamra and W Belzig, Phys. Rev. Lett. 119, 197201 (2017)
Spin pumping and shot noise in ferrimagnets: bridging ferro- and antiferromagnets

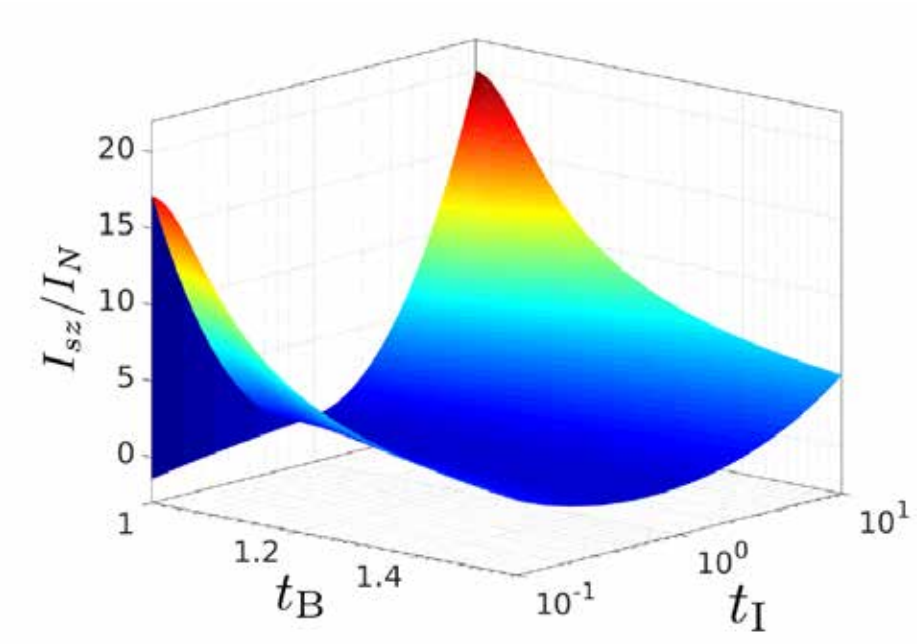
Role of asymmetry for spin pumping

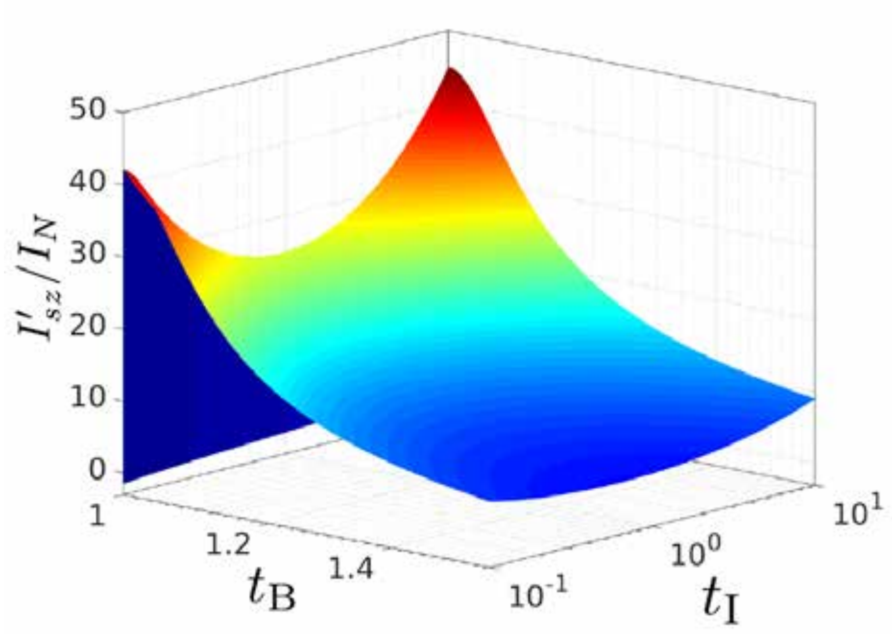


$$\frac{1}{\hbar} I_{sz} = G_{AA}(\vec{m}_A \times \dot{\vec{m}}_A)_z + G_{BB}(\vec{m}_B \times \dot{\vec{m}}_B)_z + G_{AB}\,(\vec{m}_A \times \dot{\vec{m}}_B + \vec{m}_B \times \dot{\vec{m}}_A)_z$$

A. Kamra and W Belzig, Phys. Rev. Lett. 119, 197201 (2017)
Spin pumping and shot noise in ferrimagnets: bridging ferro- and antiferromagnets

Role of cross-sublattice terms





$$G_{AB} = \sqrt{G_{AA}G_{BB}}$$

By hand: $G_{AB} = 0$

A. Kamra and W. Belzig, Phys. Rev. Lett. 119, 197201 (2017).

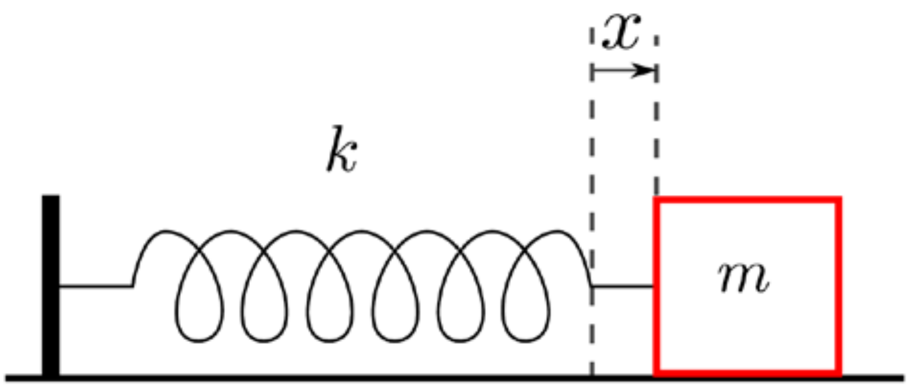
Outline

- Introduction and motivation
- Quasiparticles in ferromagnets
- Spin current shot noise and quantum of transport
- Quasiparticles in Ferrimagnets and Antiferromagnets
- Spin pumping in Ferrimagnets and Antiferromagnets
- Gilbert damping in Ferri- and Antiferromagnets

Simple dissipation model

$$\mathcal{L} = \mathcal{T} - \mathcal{U} = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$

Disspation: $$\mathcal{R} = \frac{1}{2}\Gamma\dot{x}^2$$



$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{x}} - \frac{\partial\mathcal{L}}{\partial x} + \frac{\partial\mathcal{R}}{\partial\dot{x}} = 0,$$

$$m\ddot{x} + kx + \Gamma\dot{x} = 0.$$

Rayleigh dissipation function captures viscous damping

Dynamical equation for a Ferromagnet

$$\frac{d}{dt}\frac{\delta\mathcal{L}[\mathbf{M},\dot{\mathbf{M}}]}{\delta\dot{\mathbf{M}}} - \frac{\delta\mathcal{L}[\mathbf{M},\dot{\mathbf{M}}]}{\delta\mathbf{M}} + \frac{\delta\mathcal{R}[\dot{\mathbf{M}}]}{\delta\dot{\mathbf{M}}} = 0$$

Euler-Lagrange

$$\mathcal{R}[\dot{\mathbf{M}}(\mathbf{r},t)] = \frac{\eta}{2}\int \dot{\mathbf{M}}(\mathbf{r},t)\cdot\dot{\mathbf{M}}(\mathbf{r},t)\,d\mathbf{r}$$

Rayleigh dissipation

$$\frac{\partial\mathbf{M}(\mathbf{r},t)}{\partial t} = \gamma\mathbf{M}(\mathbf{r},t)\times\left[\mathbf{H}(\mathbf{r},t) - \eta\frac{\partial\mathbf{M}(\mathbf{r},t)}{\partial t}\right]$$

Landau-Lifshitz-Gilbert equation

- Rigorous treatment of Kinetic Energy term is tricky, and require the equations to reduce to Landau-Lifshitz equation in the absence of damping
- Note: Landau-Lifshitz equation $\dot{\boldsymbol{M}} = \gamma\boldsymbol{M}\times\boldsymbol{H} - \lambda\boldsymbol{M}\times(\boldsymbol{M}\times\boldsymbol{H})$ gives $\dot{\boldsymbol{M}} \to \infty$ for $\lambda \to \infty$! Cured by reduced gyromagnetic ratio $\gamma \to \gamma^*/(1+\lambda^2)$ reducing the precession.

Rayleigh dissipation functional captures viscous damping

T. L. Gilbert. A phenomenological theory of damping in ferromagnetic materials. IEEE Transactions on Magnetics 40, 3443 (2004).

Two-sublattice magnet

$$\frac{d}{dt}\frac{\delta \mathcal{L}[\cdot]}{\delta \dot{\boldsymbol{M}}_{A,B}} - \frac{\delta \mathcal{L}[\cdot]}{\delta \boldsymbol{M}_{A,B}} = -\frac{\delta R[\dot{\boldsymbol{M}}_A, \dot{\boldsymbol{M}}_B]}{\delta \dot{\boldsymbol{M}}_{A,B}}$$

$$R[\dot{\boldsymbol{M}}_A, \dot{\boldsymbol{M}}_B] = \int_V d^3r \left(\frac{\eta_{AA}}{2} \dot{\boldsymbol{M}}_A \cdot \dot{\boldsymbol{M}}_A + \frac{\eta_{BB}}{2} \dot{\boldsymbol{M}}_B \cdot \dot{\boldsymbol{M}}_B + \eta_{AB} \dot{\boldsymbol{M}}_A \cdot \dot{\boldsymbol{M}}_B \right)$$

$$\dot{\boldsymbol{M}}_A = -|\gamma_A| \left(\boldsymbol{M}_A \times \mu_0 \boldsymbol{H}_A \right) + |\gamma_A| \eta_{AA} \left(\boldsymbol{M}_A \times \dot{\boldsymbol{M}}_A \right) + |\gamma_A| \eta_{AB} \left(\boldsymbol{M}_A \times \dot{\boldsymbol{M}}_B \right)$$

$$\dot{\boldsymbol{M}}_B = -|\gamma_B| \left(\boldsymbol{M}_B \times \mu_0 \boldsymbol{H}_B \right) + |\gamma_B| \eta_{AB} \left(\boldsymbol{M}_B \times \dot{\boldsymbol{M}}_A \right) + |\gamma_B| \eta_{BB} \left(\boldsymbol{M}_B \times \dot{\boldsymbol{M}}_B \right)$$

$$\mu_0 \boldsymbol{H}_{A,B} = -\frac{\delta F[\boldsymbol{M}_A, \boldsymbol{M}_B]}{\delta \boldsymbol{M}_{A,B}}$$

A. Kamra, R. E. Troncoso, W. Belzig, and A. Brataas. Gilbert damping phenomenology for two-sublattice magnets. Phys. Rev. B 98, 184402 (2018).

Two-sublattice magnet

$$\dot{\hat{\boldsymbol{m}}}_A = -\left|\gamma_A\right| \left(\hat{\boldsymbol{m}}_A \times \mu_0 \boldsymbol{H}_A\right) + \alpha_{AA} \left(\boldsymbol{m}_A \times \dot{\hat{\boldsymbol{m}}}_A\right) + \alpha_{AB} \left(\hat{\boldsymbol{m}}_A \times \dot{\hat{\boldsymbol{m}}}_B\right)$$

$$\dot{\hat{\boldsymbol{m}}}_B = -\left|\gamma_B\right| \left(\hat{\boldsymbol{m}}_B \times \mu_0 \boldsymbol{H}_B\right) + \alpha_{BA} \left(\hat{\boldsymbol{m}}_B \times \dot{\hat{\boldsymbol{m}}}_A\right) + \alpha_{BB} \left(\hat{\boldsymbol{m}}_B \times \dot{\hat{\boldsymbol{m}}}_B\right)$$

$$\tilde{\alpha} = \begin{pmatrix} \alpha_{AA} & \alpha_{AB} \\ \alpha_{BA} & \alpha_{BB} \end{pmatrix} = \begin{pmatrix} |\gamma_A|\eta_{AA}M_{A0} & |\gamma_A|\eta_{AB}M_{B0} \\ |\gamma_B|\eta_{AB}M_{A0} & |\gamma_B|\eta_{BB}M_{B0} \end{pmatrix}$$

$$\frac{\alpha_{AB}}{\alpha_{BA}} = \frac{|\gamma_A|M_{B0}}{|\gamma_B|M_{A0}}.$$

Gilbert damping matrix

A. Kamra, R. E. Troncoso, W. Belzig, and A. Brataas. Gilbert damping phenomenology for two-sublattice magnets. Phys. Rev. B 98, 184402 (2018).

Collinear Ground State

$$F[\boldsymbol{M}_A, \boldsymbol{M}_B] = \int_V d^3r \left[-\mu_0 H_0 (M_{Az} + M_{Bz}) - K_A M_{Az}^2 - K_B M_{Bz}^2 + J \boldsymbol{M}_A \cdot \boldsymbol{M}_B\right]$$

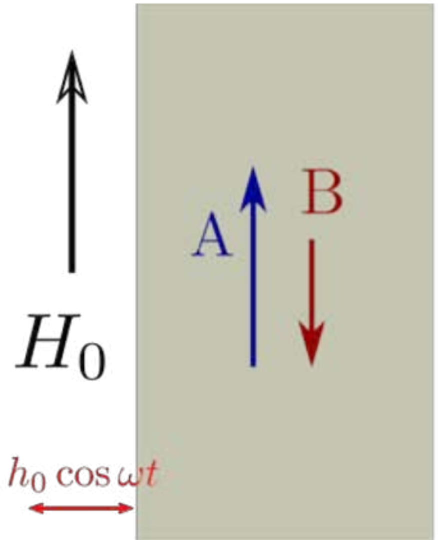
Resonance frequencies:

$$\omega_{r\pm} = \frac{\pm(\Omega_A - \Omega_B) + \sqrt{(\Omega_A + \Omega_B)^2 - 4J^2|\gamma_A||\gamma_B|M_{A0}M_{B0}}}{2}$$

Line widths:

$$\frac{\omega_{i\pm}}{\omega_{r\pm}} = \frac{\bar{\alpha}\,(\Omega_A + \Omega_B) - 2J|\gamma_B|M_{A0}\alpha_{AB}}{\omega_{r+} + \omega_{r-}} \pm \Delta\bar{\alpha}$$

$$\bar{\alpha} \equiv (\alpha_{AA} + \alpha_{BB})/2 \qquad \Delta\bar{\alpha} \equiv (\alpha_{AA} - \alpha_{BB})/2$$



- Renormalization of resonance width by off-diagonal damping $\alpha_{AB} > 0$
- Difference resonance widths due to different sublattice dampings $\alpha_{AA} \neq \alpha_{BB}$

A. Kamra, R. E. Troncoso, W. Belzig, and A. Brataas. Gilbert damping phenomenology for two-sublattice magnets. Phys. Rev. B 98, 184402 (2018).

Compensated Ferrimagnets

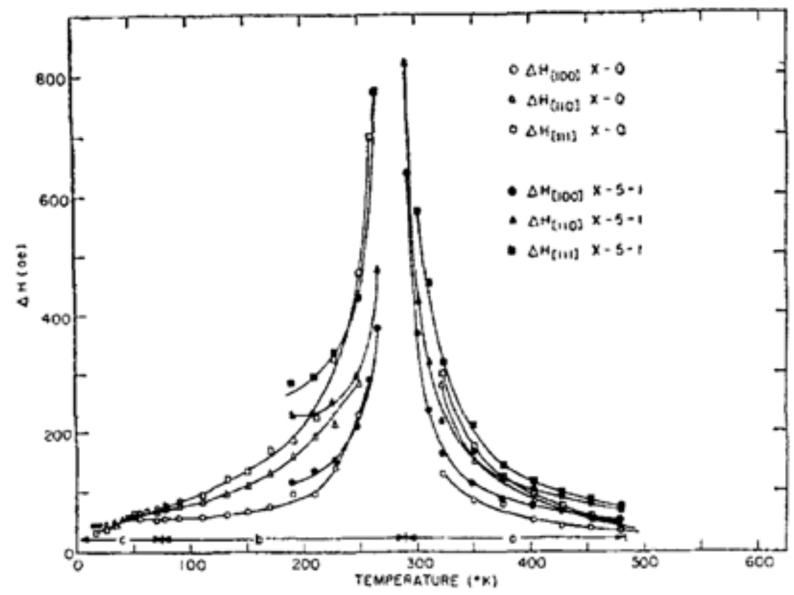
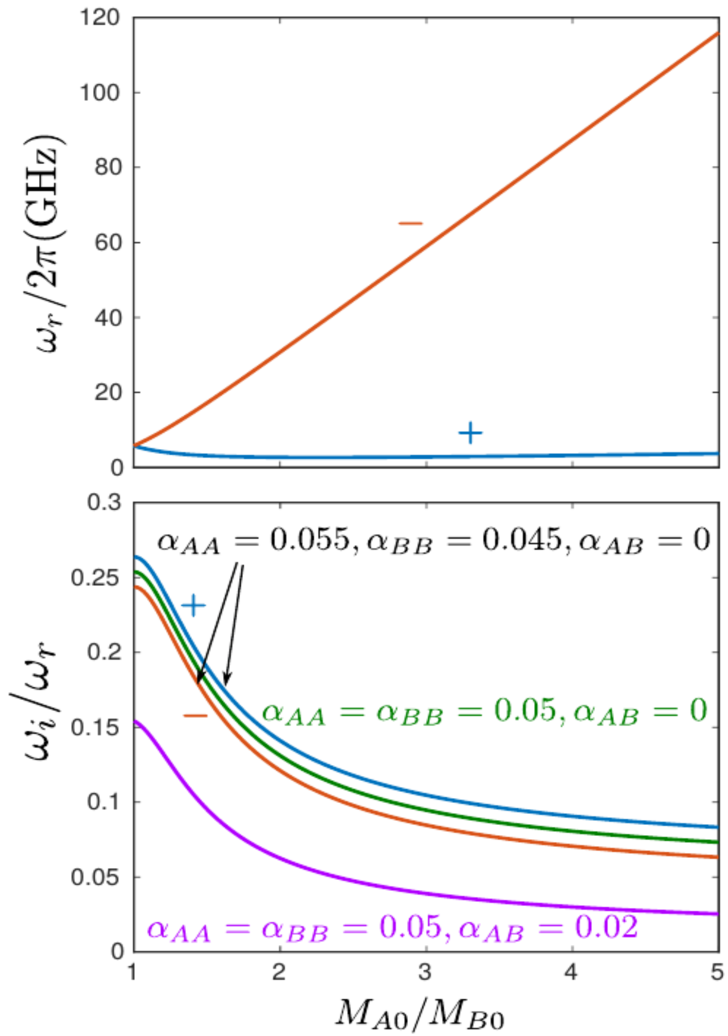


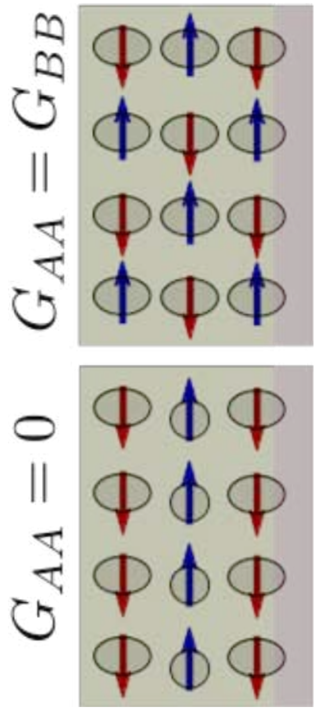
FIG. 5. Temperature variation of ΔH in GdIG.

G. P. Rodrigue, H. Meyer, and R. V. Jones. Resonance measurements in magnetic garnets. J. Appl. Phys. 31, S376 (1960).



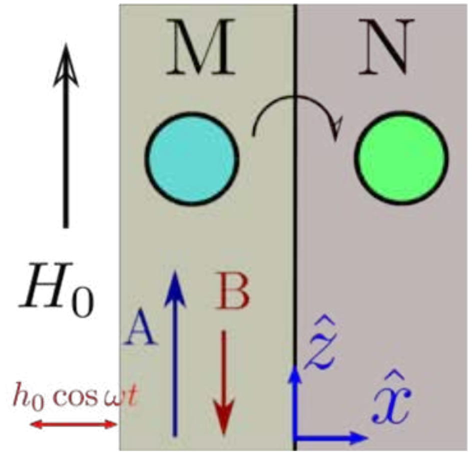
A. Kamra, R. E. Troncoso, W. Belzig, and A. Brataas. Gilbert damping phenomenology for two-sublattice magnets. Phys. Rev. B 98, 184402 (2018).

Spin pumping mediated damping



$$\boldsymbol{I}_s = \frac{\hbar}{e} \sum_{i,j=\{A,B\}} G_{ij} \left(\hat{\boldsymbol{m}}_i \times \dot{\hat{\boldsymbol{m}}}_j \right)$$

$$\alpha'_{ij} = \frac{\hbar G_{ij} |\gamma_i|}{e M_{i0} V}$$



$$\frac{\omega_{i\pm}}{\omega_{r\pm}} = \frac{\bar{\alpha}\left(\Omega_A + \Omega_B\right) - 2J|\gamma_B| M_{A0} \alpha_{AB}}{\omega_{r+} + \omega_{r-}} \pm \Delta\bar{\alpha}$$

Summary:

- A two-sublattice ferromagnet is not described by a simple ferromagnet
- non-diagonal damping has observable but difficult in collinear magnets
- No need for artificially enhanced damping around the compensation point

Summary: aspects of quantum magnonics

- Dipolar interaction-mediated squeezing and hybridi-zation of magnons with spin greater (lesser) than $\hbar$
- Spin current-shot noise as probe of non-integer spins
- Spin-zero excitations in Ferri- and Antiferromagnets
- Importance of interface/damping asymmetry and cross terms for spin pumping and spin shot noise in Fi/AF-N

- **A. Kamra** and W. Belzig,
  Super-Poissonian shot noise of squeezed-magnon mediated spin transport
  Phys. Rev. Lett. **116**, 146601 (2016).
- **A. Kamra** and W. Belzig,
  Magnon-mediated spin current noise in ferromagnet|nonmagnetic conductor hybrids,
  Phys. Rev. B **94**, 014419 (2016).
- **A. Kamra**, U. Agrawal, and W. Belzig
  Noninteger-spin magnonic excitations in untextured magnets
  Phys. Rev. B, **96**, 020411(R) (2017).
- **A. Kamra** and W. Belzig
  Spin pumping and shot noise in ferrimagnets: bridging ferro- and antiferromagnets,
  Phys. Rev. Lett. **119**, 197201 (2017)
- **A. Kamra**, R. E. Troncoso, W. Belzig, and A. Brataas
  Gilbert damping phenomenology for two-sublattice magnets.
  Phys. Rev. B **98**, 184402 (2018).