# A Josephson relation for e/3 and e/5 fractionally charged with anyons


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**OPEN POSITION**
for 18-24 months
**Post-doct.**
(urgent)

AGENCE NATIONALE DE LA RECHERCHE
ANR

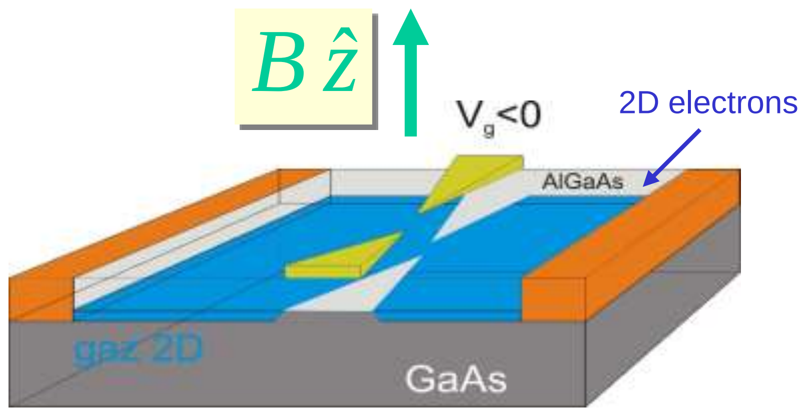
# OUTLINE

- Quantum Hall edge states and Fractional Quantum Hall Effect
- PHOTON-ASSISTED TRANSPORT
    - Photon-assisted processes
    - A JOSEPHSON Relation for Photon Assisted Shot Noise (PASN)
- Experimental Results
    - e*=e/3
    - e*=e/5
- CONCLUSION and PERSPECTIVES

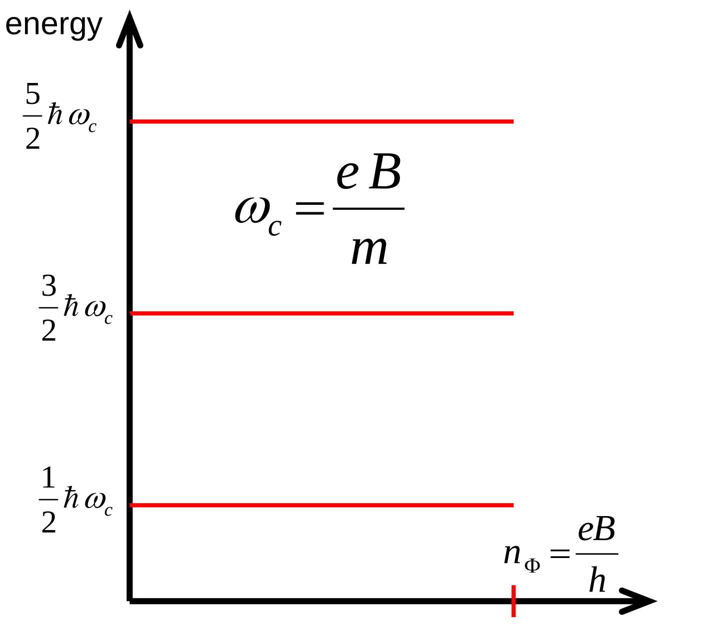
$$f_{J.} = \frac{e^*V}{h}$$

X. G. Wen (1991)

# Quantum Hall Effect (QHE)




III-V semi-conductor heterojunction GaAs/GaAlAs



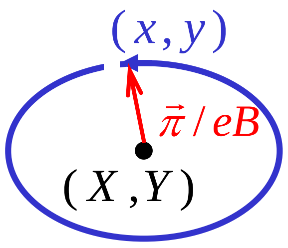

$$R_{Hall} = \frac{B}{e\,n_s} = \frac{h}{e^2}\,\frac{1}{(\nu = k)}$$

$$H = \frac{1}{2m}\left(\vec{p} + e\vec{A}\right)^2 = \frac{\vec{\pi}^2}{2m}$$

$$\left[\pi_x, \pi_y\right] = -i\hbar eB \longrightarrow E_n = \hbar\omega_c\left(n + \frac{1}{2}\right)$$

$$X = x - \frac{\pi_y}{eB}$$

$$Y = y + \frac{\pi_x}{eB}$$



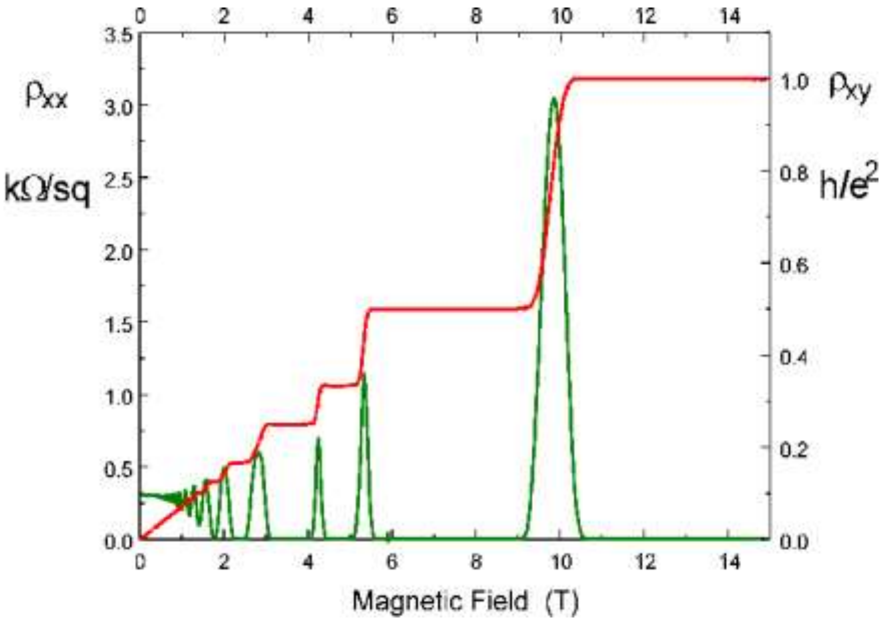

cyclotron motion

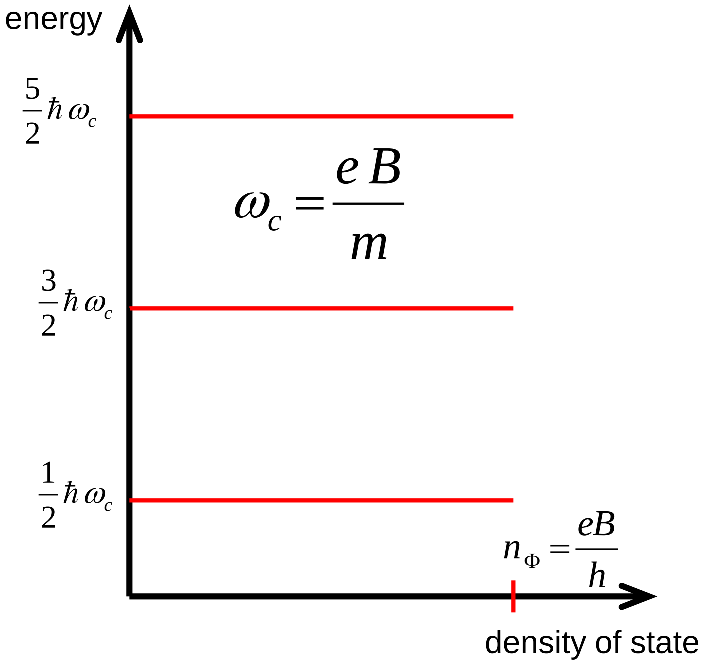
$$[X, Y] = -i\frac{\hbar}{eB} \longrightarrow B\,\Delta X.\Delta Y = \frac{h}{e}$$

cyclotron motion is frozen → 1D dynamics

# Integer Quantum Hall Effect (IQHE)

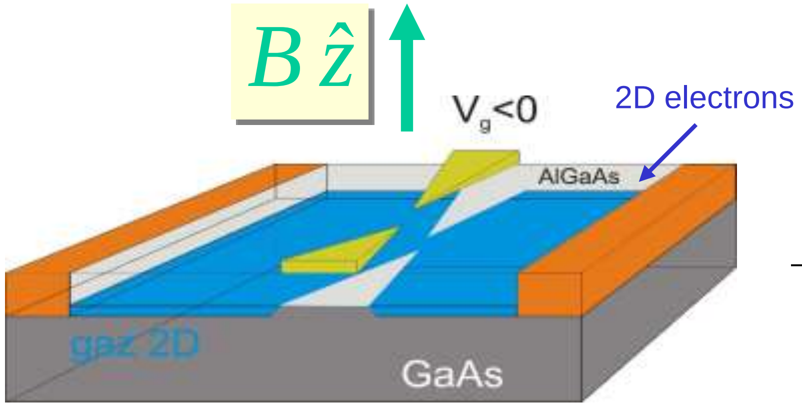
$R_{hall}=(h/e^2)1/\nu \quad \nu=1,2,3,\ldots$



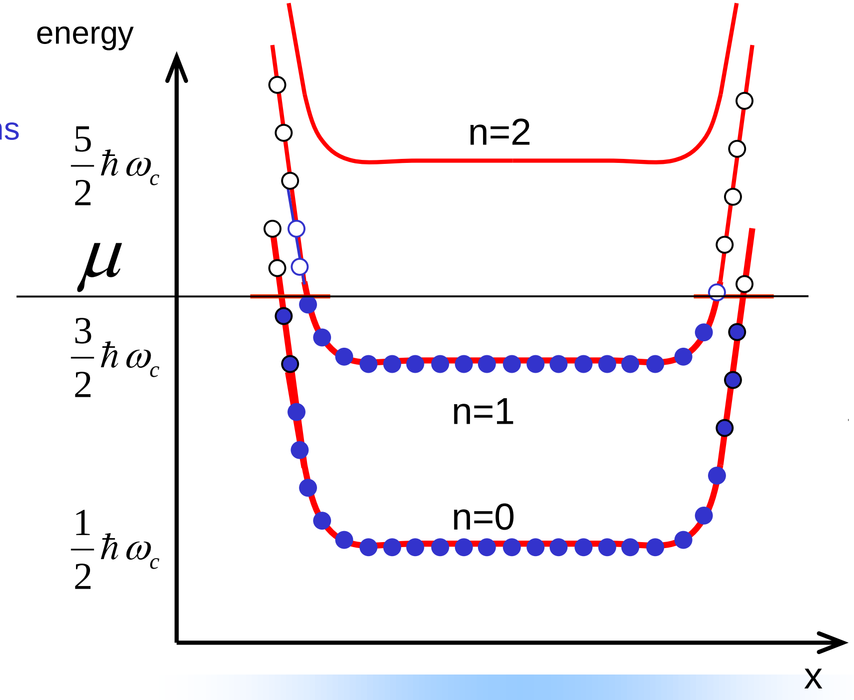



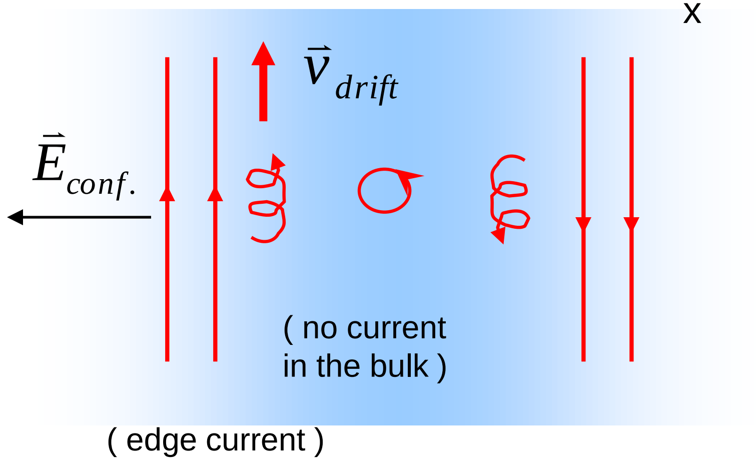

$$R_{Hall}=\frac{B}{e\,n_s}=\frac{h}{e^2}\frac{1}{(\nu=k)}$$

# QHE and EDGE STATES




III-V semi-conductor heterojunction GaAs/GaAlAs



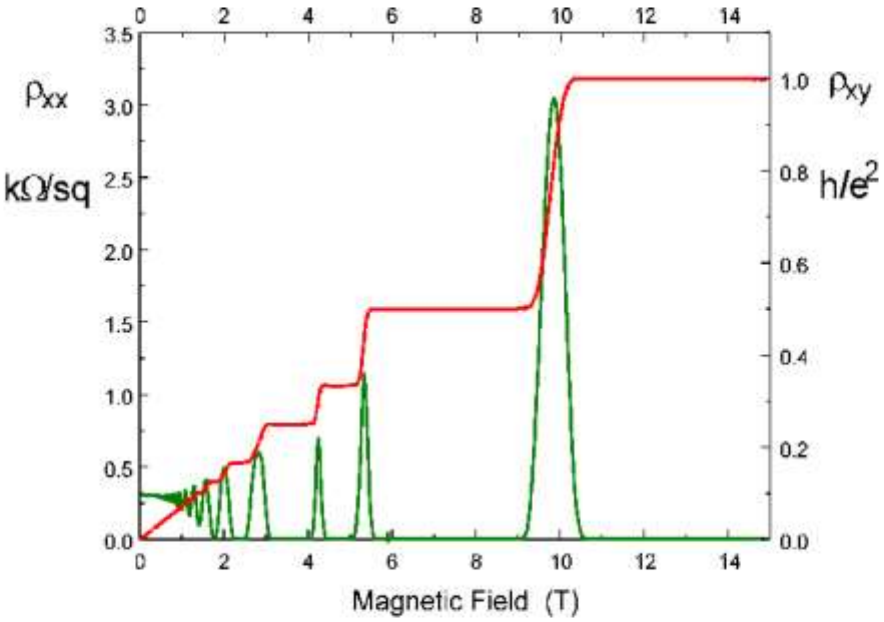




$$\vec{v}_{drift} = \frac{\vec{E}_{conf.}}{B} \times \hat{z}$$

cyclotron motion drift → chiral 1D EDGE CHANNELS

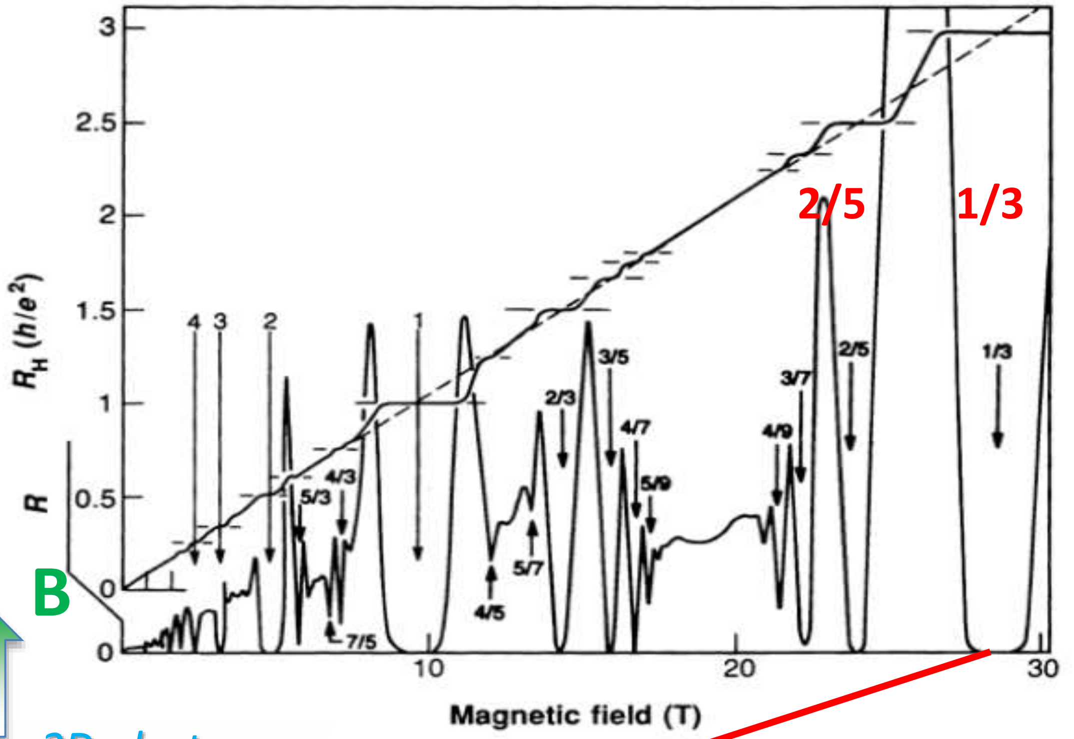
Integer Quantum Hall Effect (IQHE)

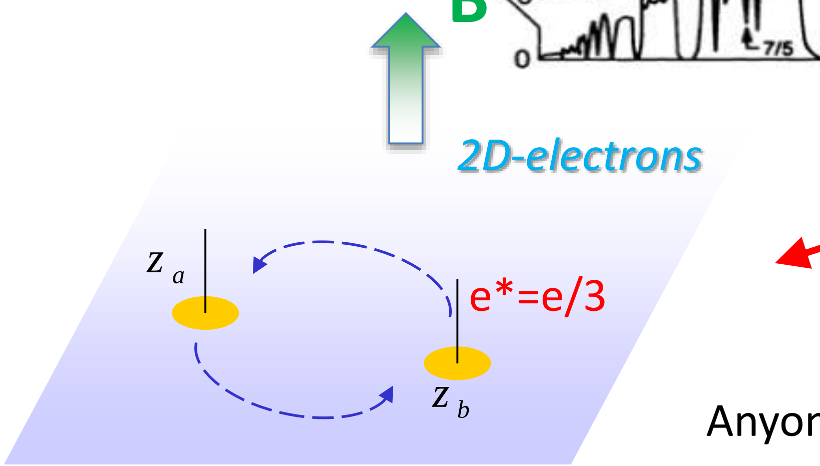
$R_{hall}=(h/e^2)1/\nu \quad \nu=1,2,3, \ldots$




Fractional Quantum Hall Effect (FQHE)

$R_{hall}=(h/e^2)1/\nu \quad \nu=1/3,2/5,3/7, \ldots 2/3, 3/5, 4/7, \ldots$




Anyons $\Psi(a,b)=e^{i\theta}\,\Psi(b,a) \quad \theta=2\pi/3$

# DC SHOT NOISE: Integer QHE

$$S_I = 2eI_0\,D(1-D)$$

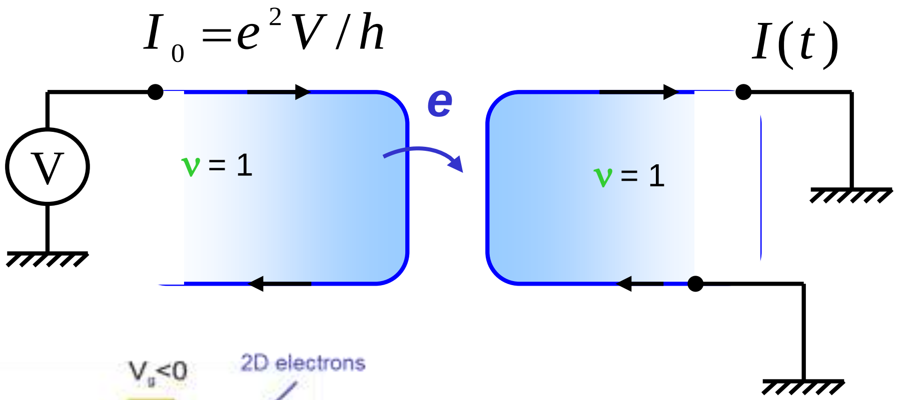
G. B. Lesovik, JETP Letters49, 594 (1989)

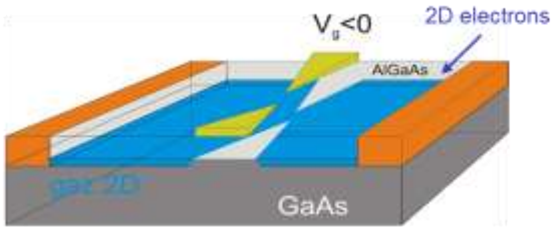
$$I_0 = e^2V/h$$

$$I_0 = I + I_B$$

transmitted ($D$) reflected (1-$D$)

strong barrier :

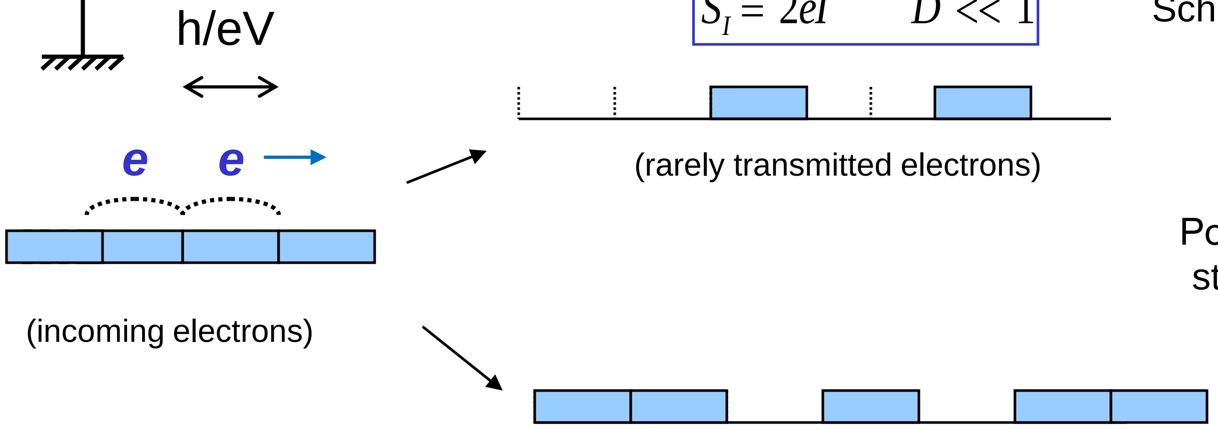



III-V semi-conductor heterojunction GaAs/GaAlAs

$S_I = 2eI \quad D \ll 1$

Schottky (1918)




(rarely transmitted electrons)

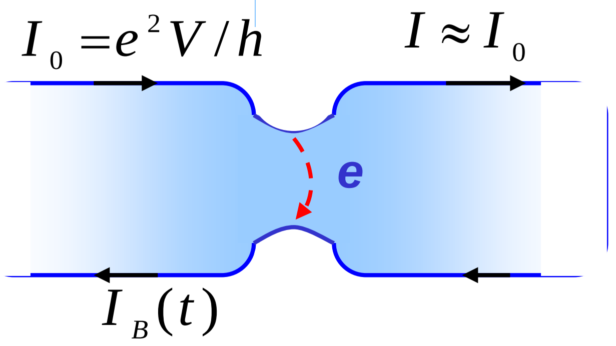
(incoming electrons)

Poisson's statistics

(rarely transmitted holes)

$S_I = 2eI_B \quad D \approx 1$

weak barrier :

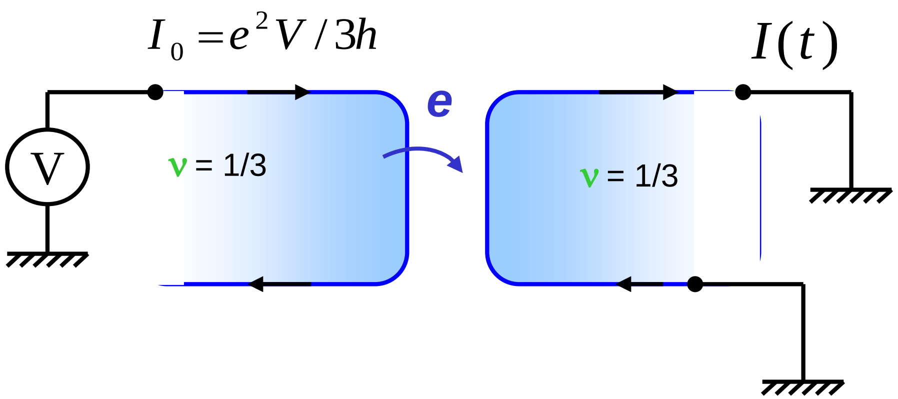


# DC SHOT NOISE: FQHE

$$I_0 = e^2 V / 3h$$
$$I_0 = I + I_B$$

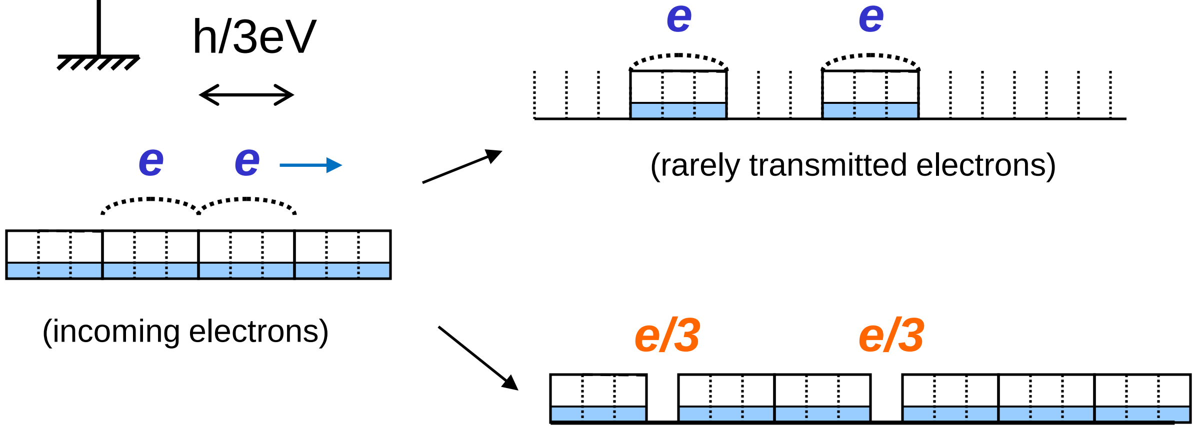
transmitted ($D$) reflected (1-$D$)

strong barrier :




ν=1/3 Laughlin state

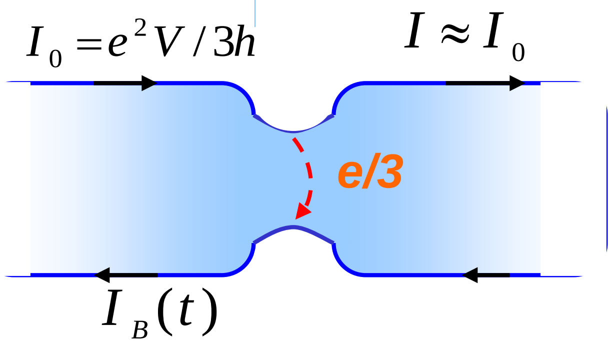
$$S_I = 2eI \qquad D << 1$$




$$S_I = 2\frac{e}{3} I_B \qquad D \approx 1$$

First observation:
*CEA Saclay 1997*
*Weizmann 1997*

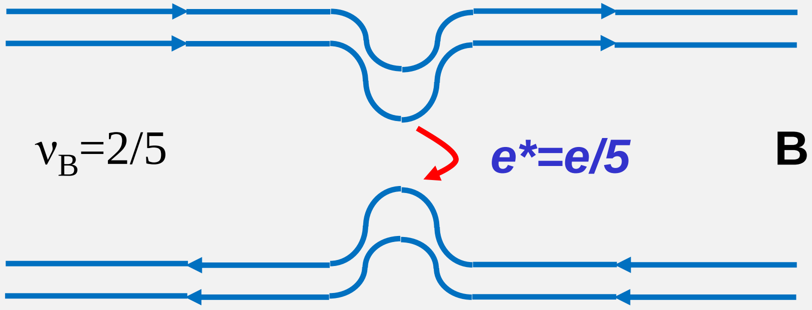
weak barrier :



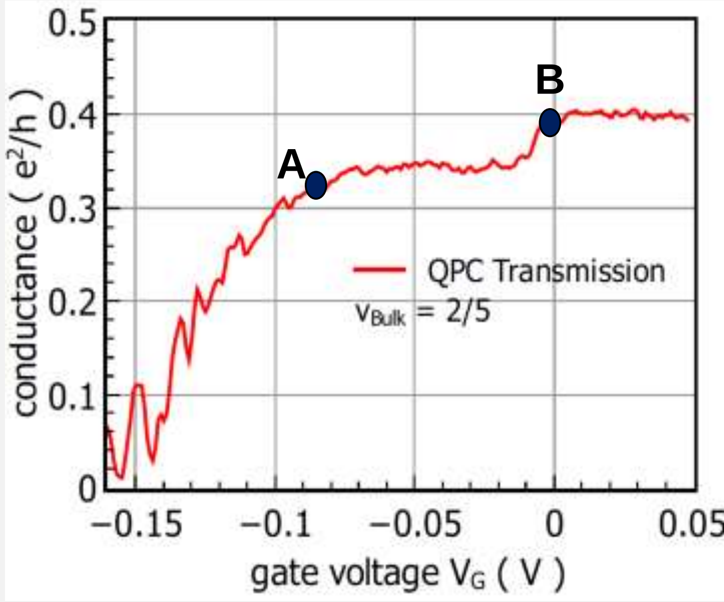

derived from chiral-Luttinger liquid approach
*( X.G. Wen 1995, C. Kane + M. Fisher 1994; Fendley, Ludwig + Saleur (1995))*

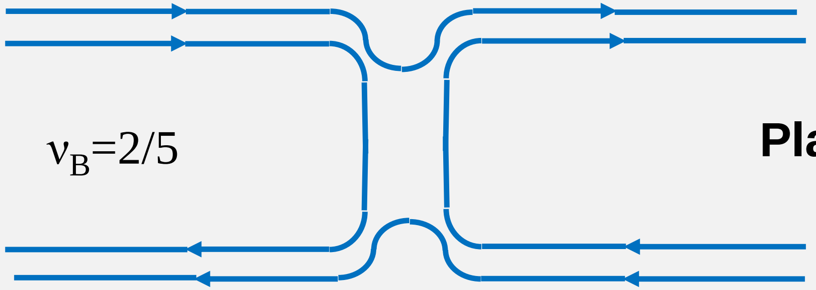
# Tunneling through a ν=2/5 Jain FQHE state

| FQHE | → | C-F. IQHE |
|---|---|---|
| ν = 1/3 | → | ν=1 |
| ν = 2/5 | → | ν=2 |
| ν = 3/7 | → | ν=3 |
| …. | → | …. |

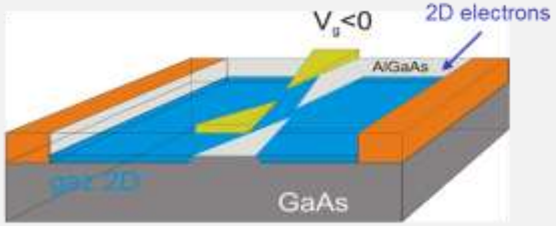



**B**: WB in 2/5

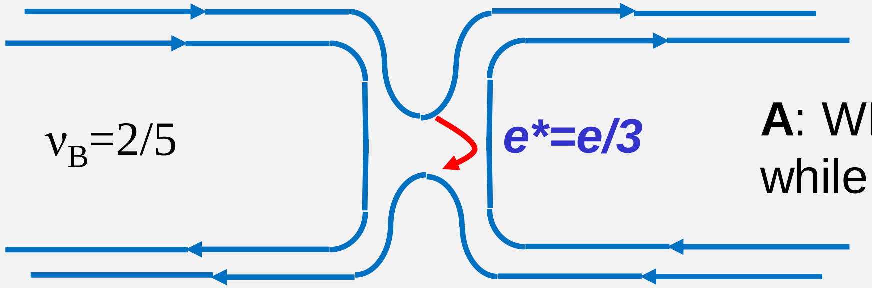



**Plateau** 1/3




III-V semi-conductor heterojunction GaAs/GaAlAs




**A**: WB in 1/3
while 2/5 reflected

J. K. Jain *Composite-fermion approach for the fractional quantum Hall effect*. Phys. Rev. Lett. 63, 199-202 (1989)

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  - Photon-assisted processes
  - A JOSEPHSON Relation for Photon Assisted Shot Noise (PASN)
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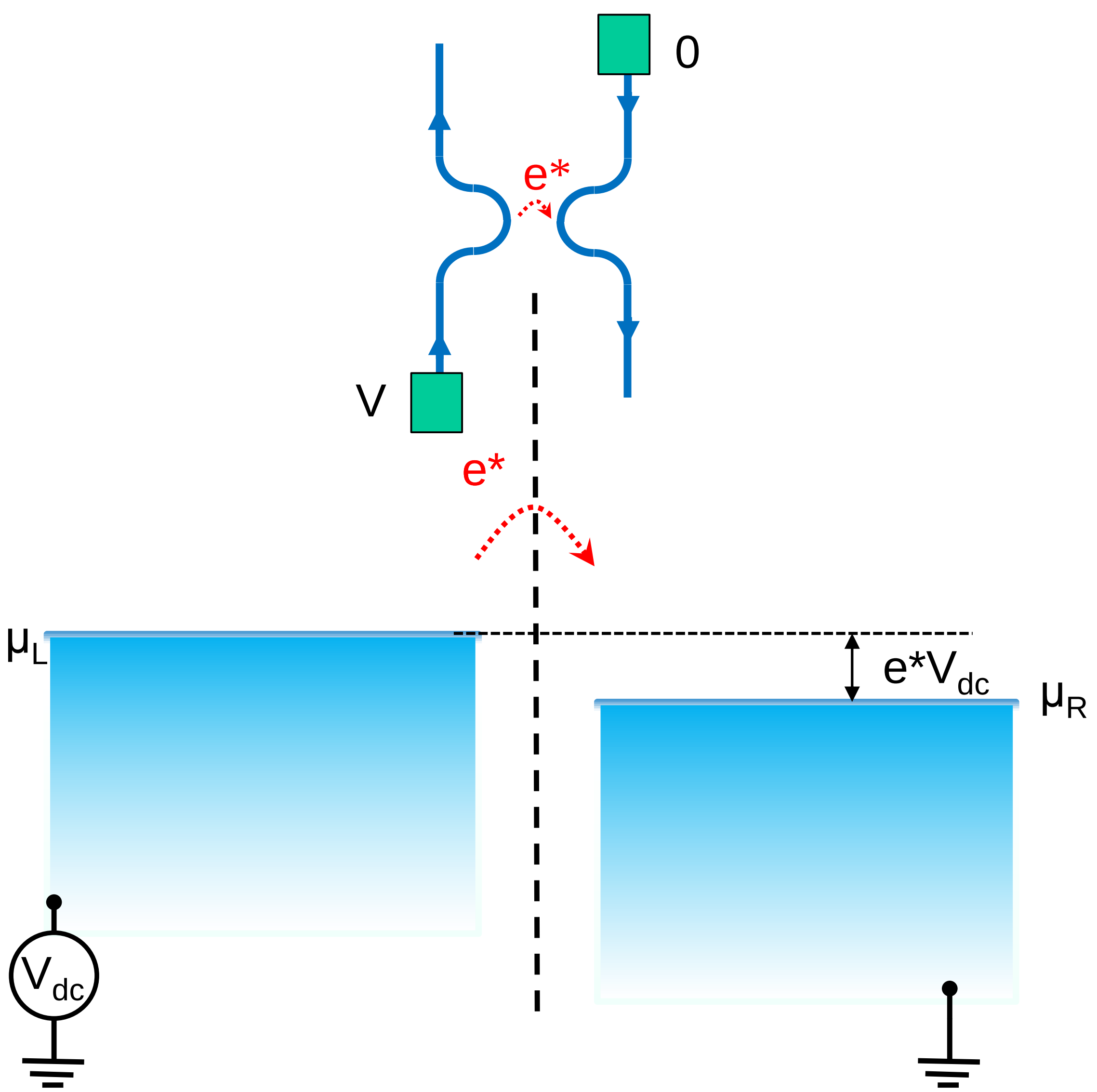
$$f_J = \frac{e^*V}{h}$$

X. G. Wen (1991)

# DC Bias transport (weak coupling)

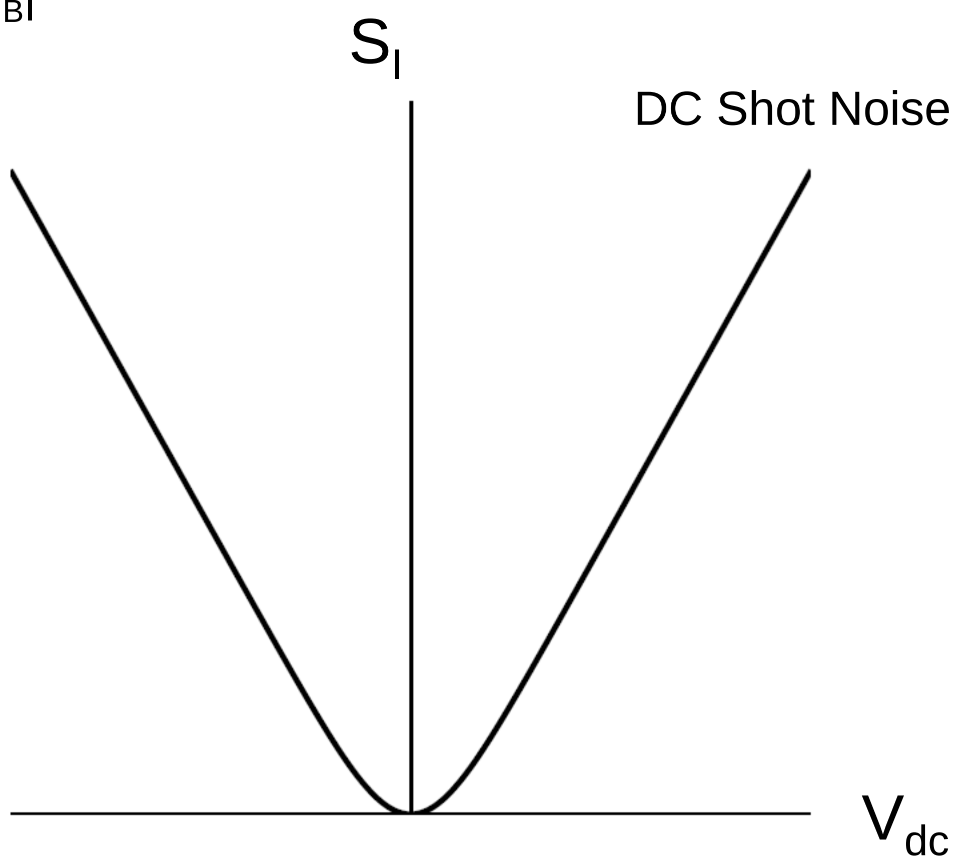
DC Transport




$\mu_R - \mu_L = e^*V_{dc}$

$I_B(V_{dc})$

$S_I^{DC} = 2e^* |I_B|$

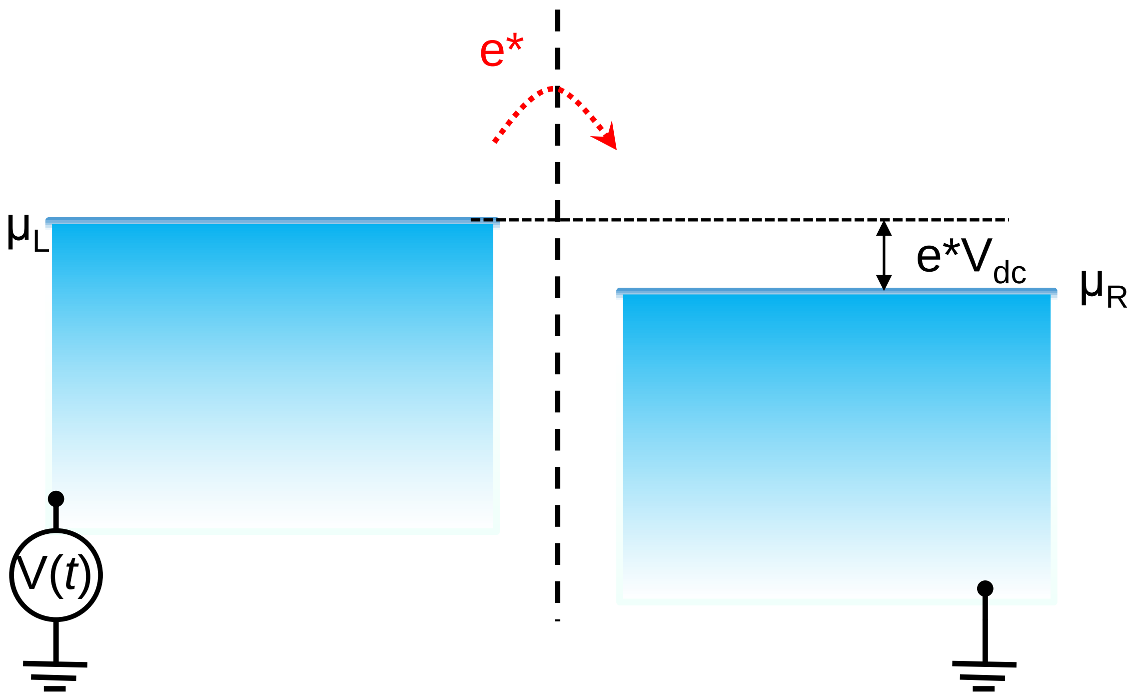


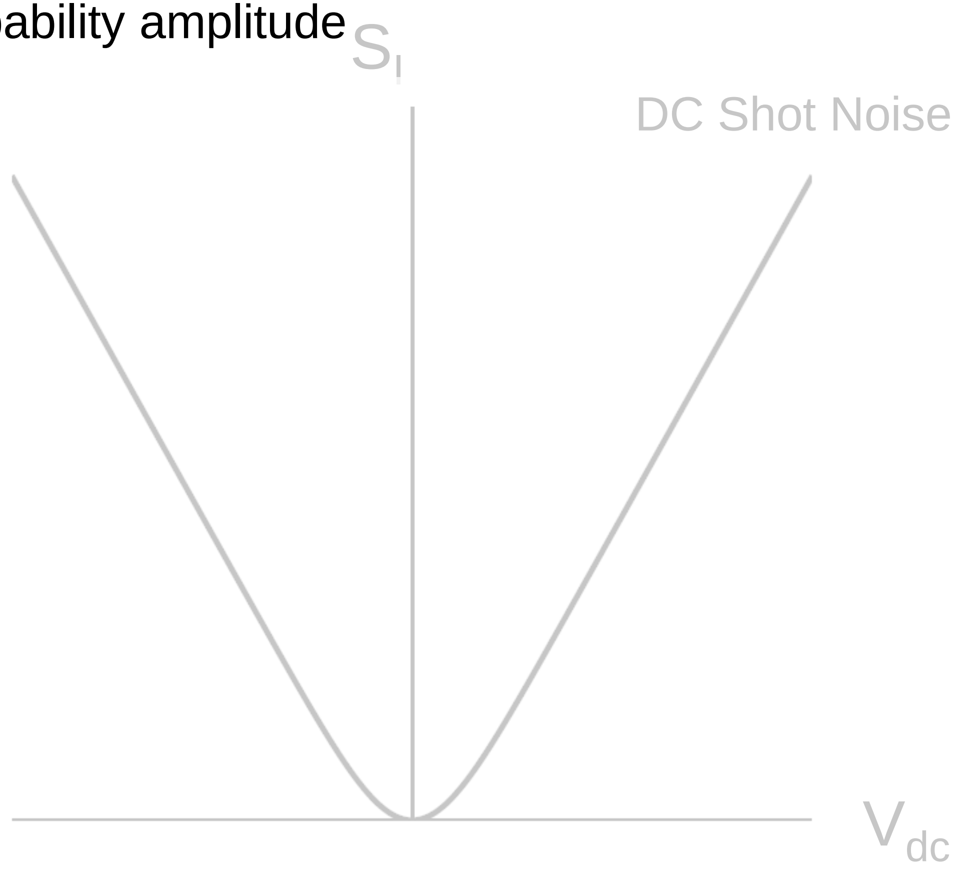
# Photon-Assisted transport (weak coupling)

$V(t)=V_{dc} + V_{ac}\cos(2\pi f t)$ $H_L \rightarrow H_L + e^* V_{ac}\cos(2\pi f t)$

→ all carriers get extra time dependent phase: $\phi(t)=\frac{1}{\hbar}\int_{-\infty}^{t} e^* V_{ac}(t')dt'$

with : $\exp(-i\phi(t)) = \sum_l p_l e^{-i2\pi l f t}$ $p_l$: photo-absorption (Floquet) probability amplitude



# Photon-Assisted Shot Noise (PASN)

$V(t)=V_{dc} + V_{ac}\cos(2\pi f t)$ $\qquad$ $H_L \rightarrow H_L + e^*V_{ac}\cos(2\pi f t)$

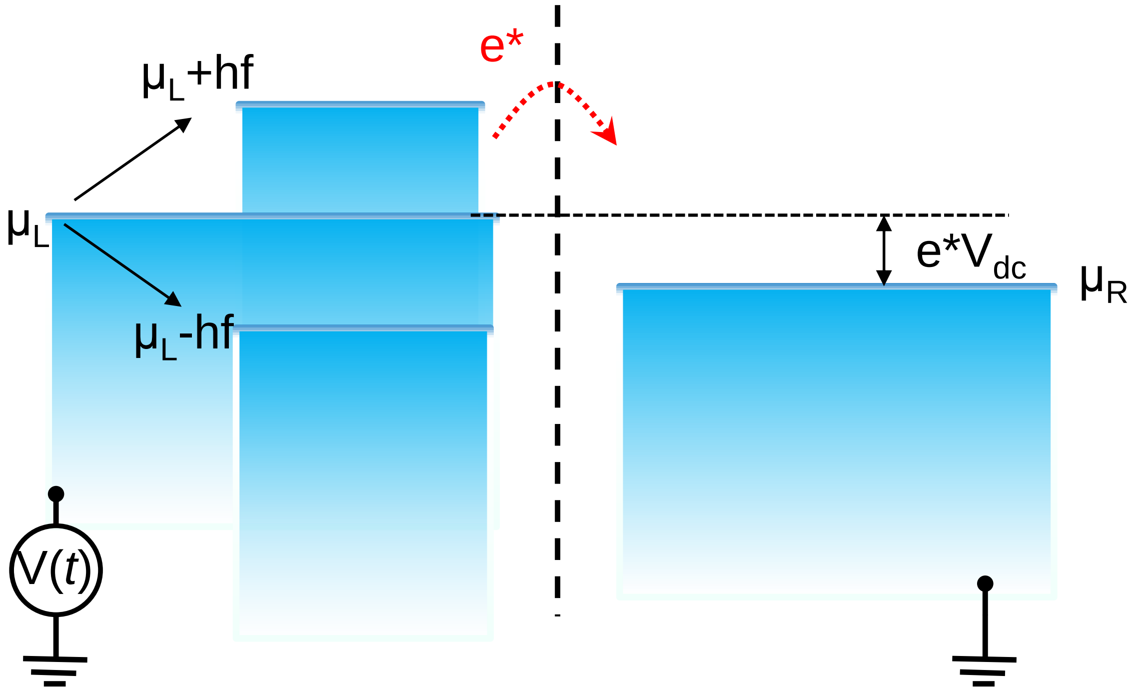
$$S_I^{PASN} = |p_0|^2 S_I^{DC}(V_{dc}) + |p_1|^2 S_I^{DC}(V_{dc}+hf/e^*) + |p_{-1}|^2 S_I^{DC}(V_{dc}-hf/e^*) + \dots$$

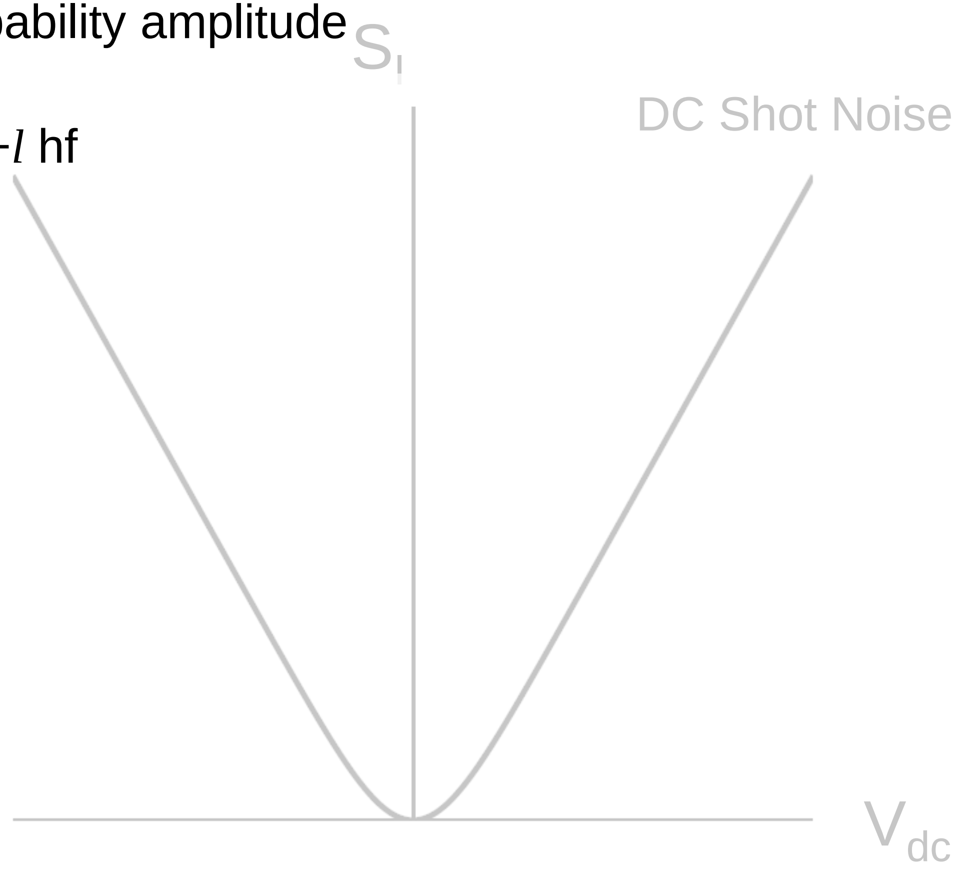
(e*=e) Lesovik and Levitov (1994)
(e*=e/m) Chamon and Wen (1995)

→ all carriers get extra time dependent phase: $\phi(t) = \frac{1}{\hbar}\int_{-\infty}^{t} e^* V_{ac}(t')dt'$

with : $\exp(-i\phi(t)) = \sum_l p_l e^{-i2\pi l f t}$ $\qquad$ $p_l$: photo-absorption (Floquet) probability amplitude

global energy scattering for all left carrier energies ε shifted by ε → ε+$l$ hf



# Photon-Assisted Shot Noise (PASN)

$V(t)=V_{dc} + V_{ac}\cos(2\pi f t)$ $\qquad H_L \rightarrow H_L + e^*V_{ac}\cos(2\pi f t)$

$S_I^{PASN} = |p_0|^2 S_I^{DC}(V_{dc}) + |p_1|^2 S_I^{DC}(V_{dc}+hf/e^*) + |p_{-1}|^2 S_I^{DC}(V_{dc}-hf/e^*) + \ldots$

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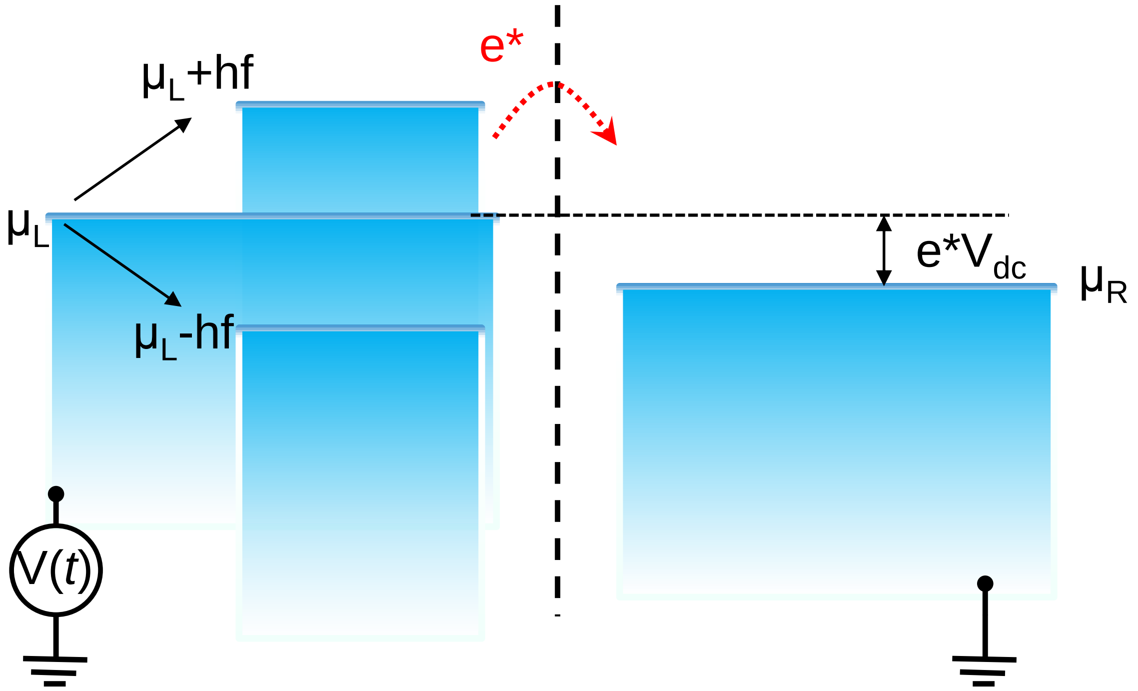
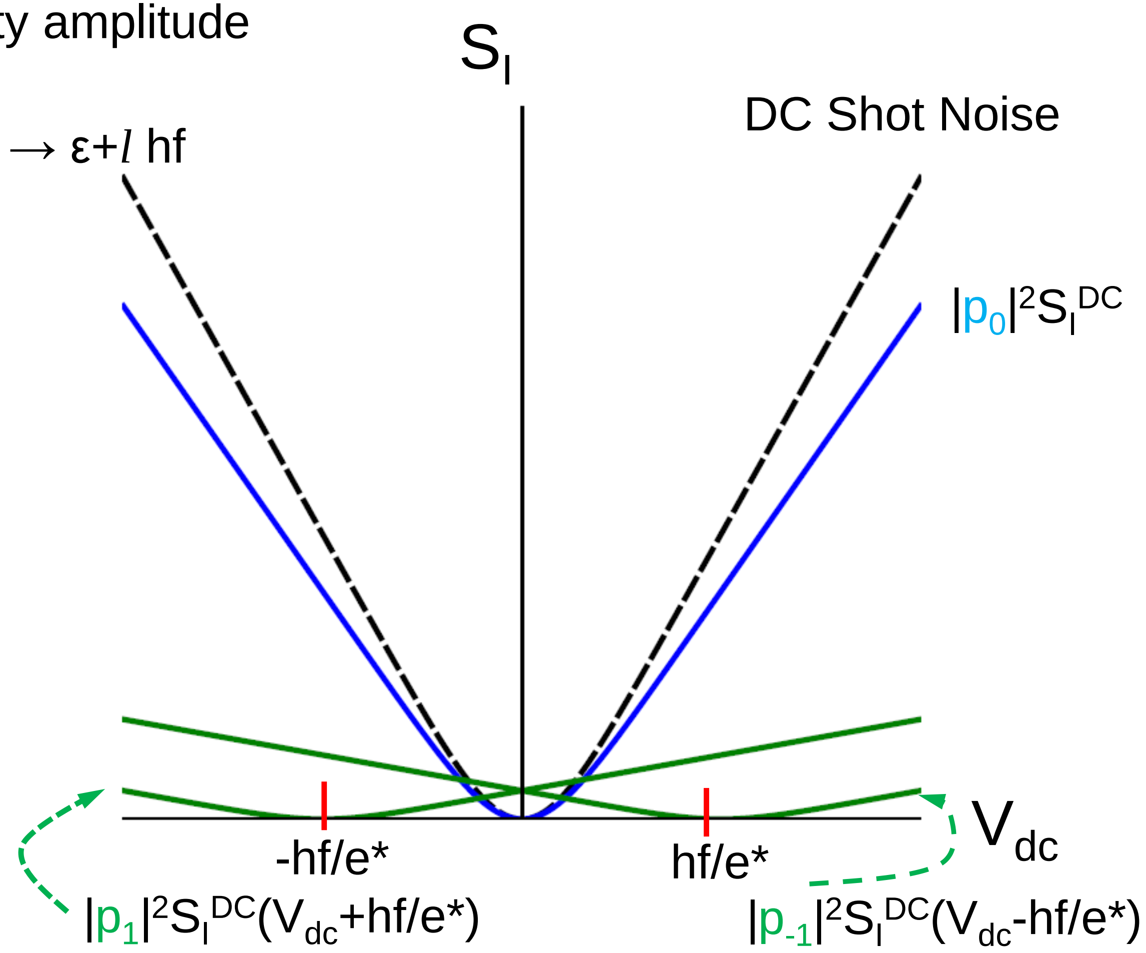
global energy scattering for all left carrier energies ε shifted by ε → ε+*l* hf



# Photon-Assisted Shot Noise (PASN)

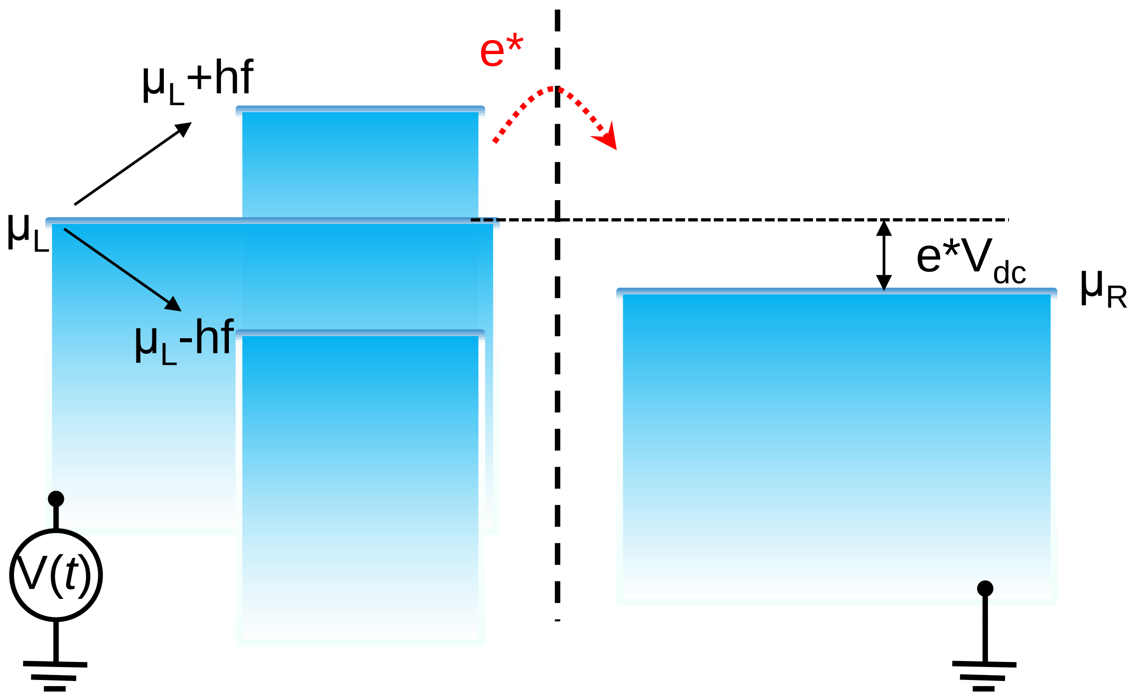
$V(t) = V_{dc} + V_{ac}\cos(2\pi f t)$

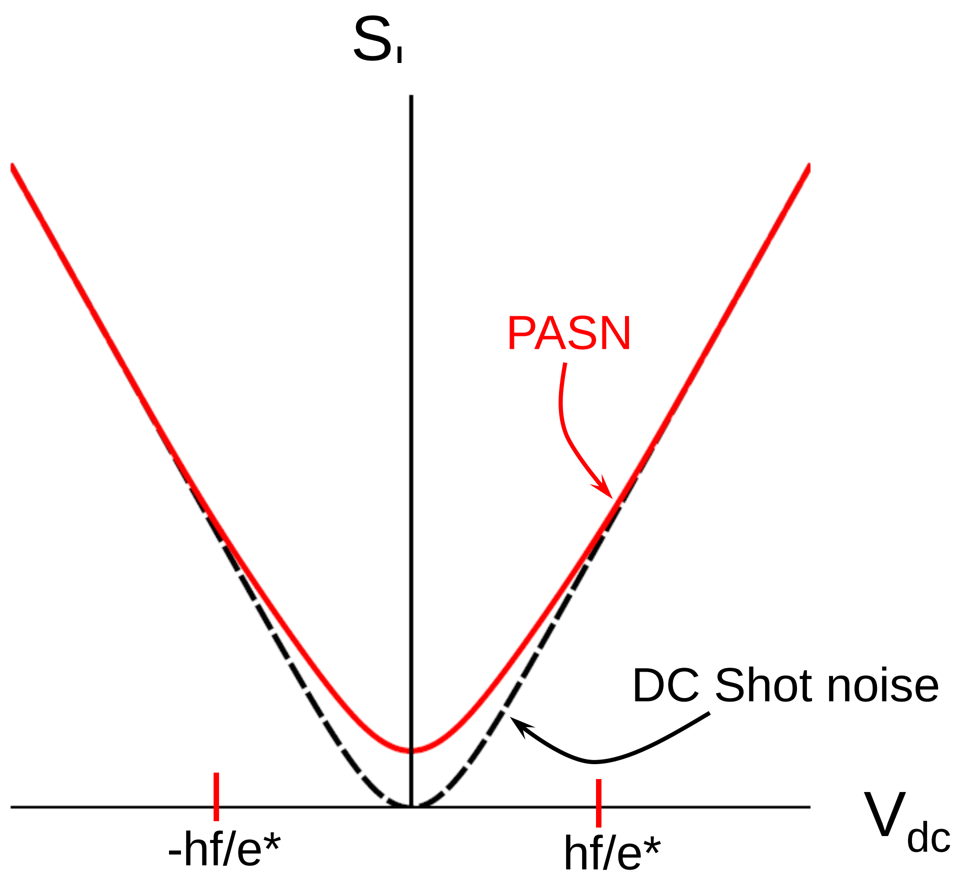
$p_l$: photo-absorption probability amplitude

$\mu_L$ shifted by $\mu_L \rightarrow \mu_L + l\,hf$ with probability $|p_l|^2$

$|p_0|^2 + |p_1|^2 + |p_{-1}|^2 + \ldots = 1$

$$S_I^{PASN} = |p_0|^2 S_I^{DC}(V_{dc}) + |p_1|^2 S_I^{DC}(V_{dc}+hf/e^*) + |p_{-1}|^2 S_I^{DC}(V_{dc}-hf/e^*) + \ldots$$



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    - Photon-assisted processes
    - A JOSEPHSON Relation for Photon Assisted Shot Noise (PASN)

$$f_{J.} = \frac{e^{*}V}{h}$$

X. G. Wen (1991)

- Experimental Results
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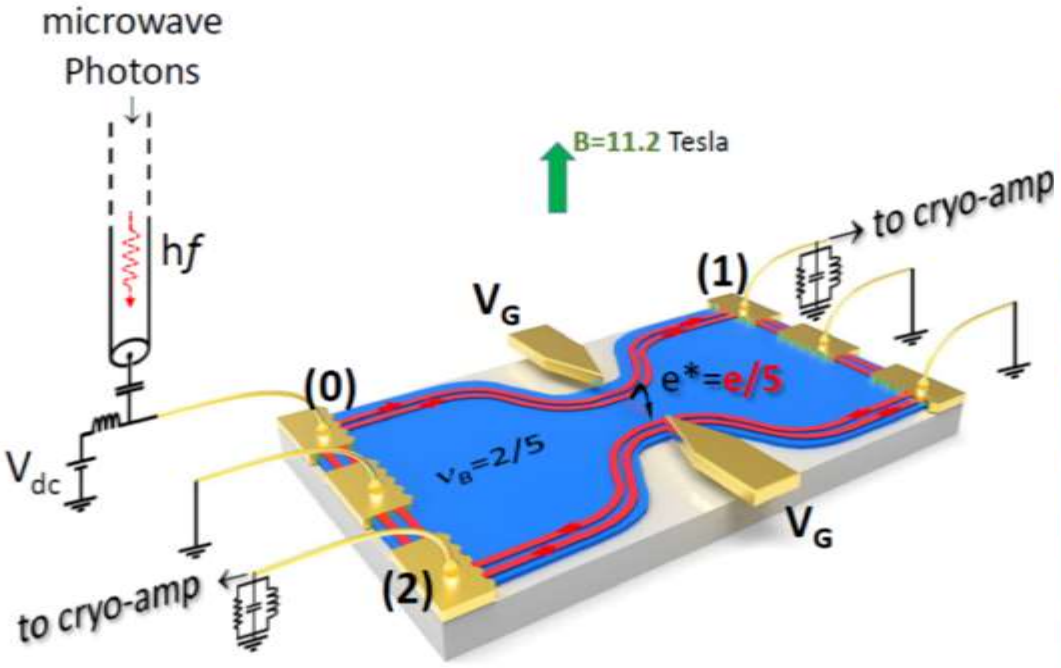
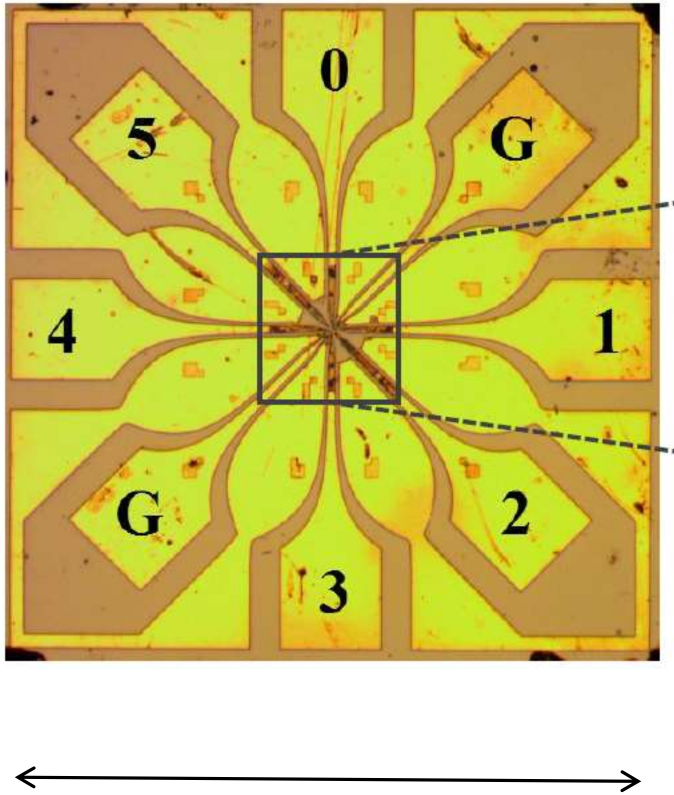
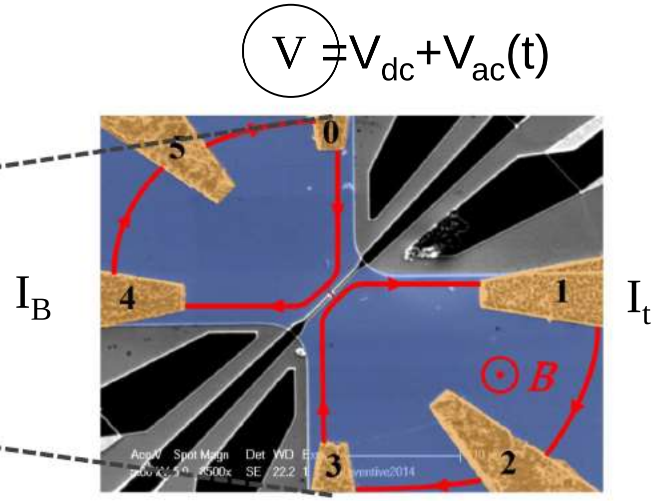
- CONCLUSION and PERSPECTIVES

# Experimental Set-up and samples

Samples: $n_s$ =1.07 $10^{11}$ $cm^{-2}$  $\mu$=3 $10^6$ $cm^2V^{-1}s^{-1}$      (from I. Farrer, D. Ritchie, Cambridge UK)



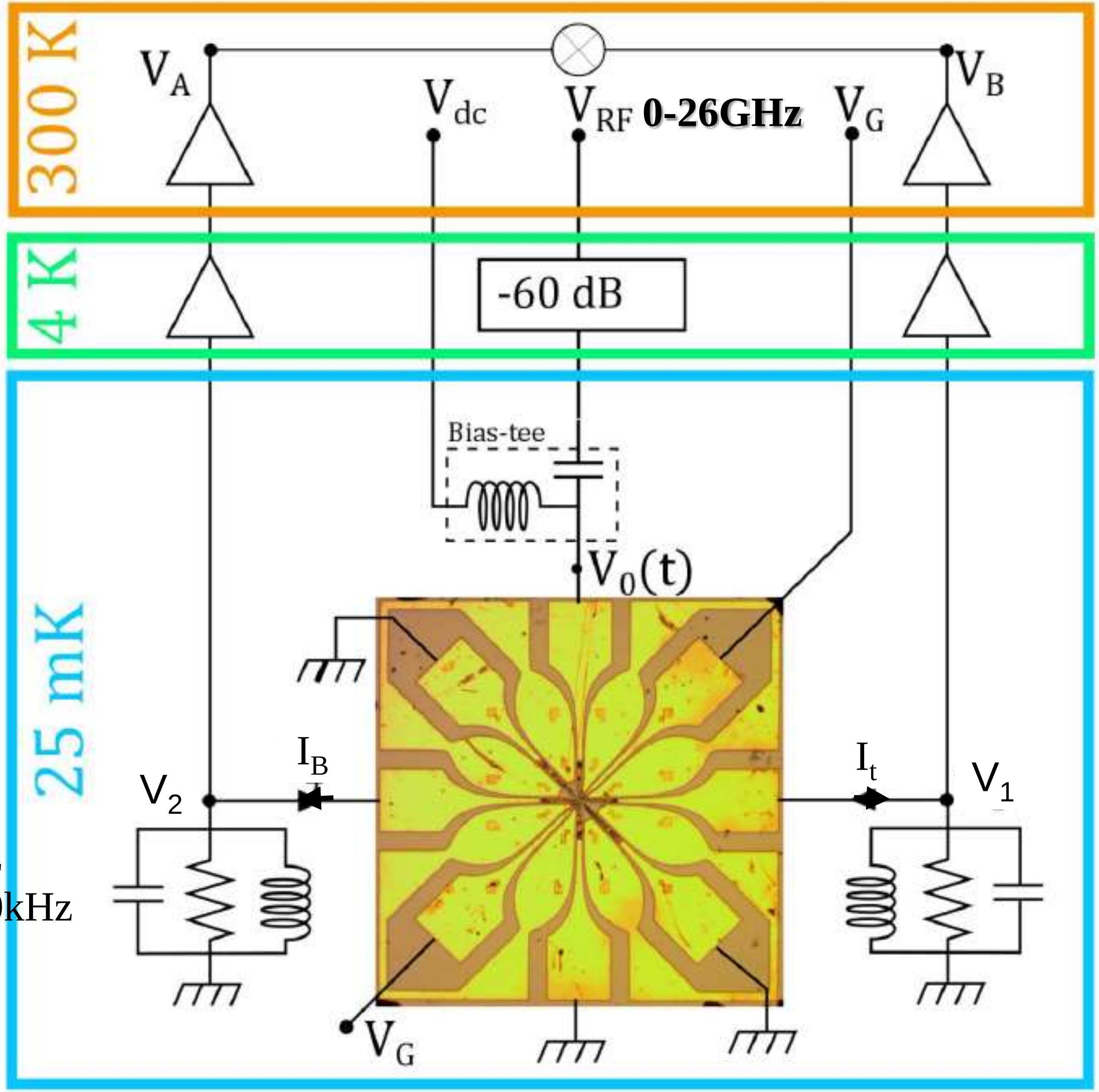




5mm

# Experimental Set-up and samples

CROSS-SPECTRUM

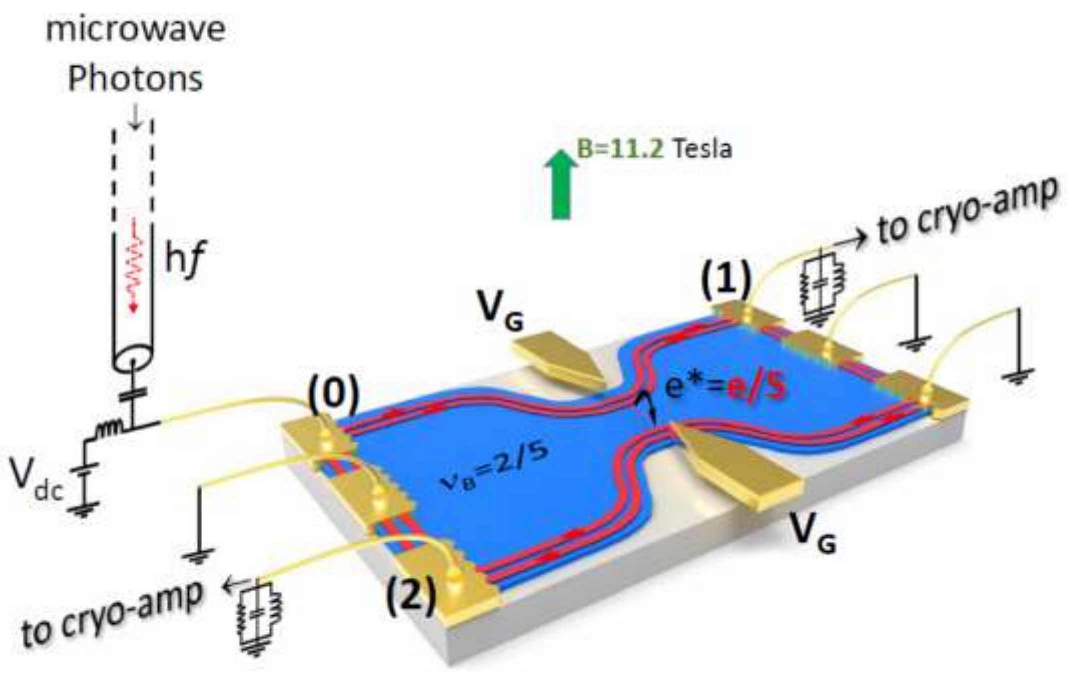
Home-made
Cryo-amp.
(**0.22nV**)$^2$/Hz

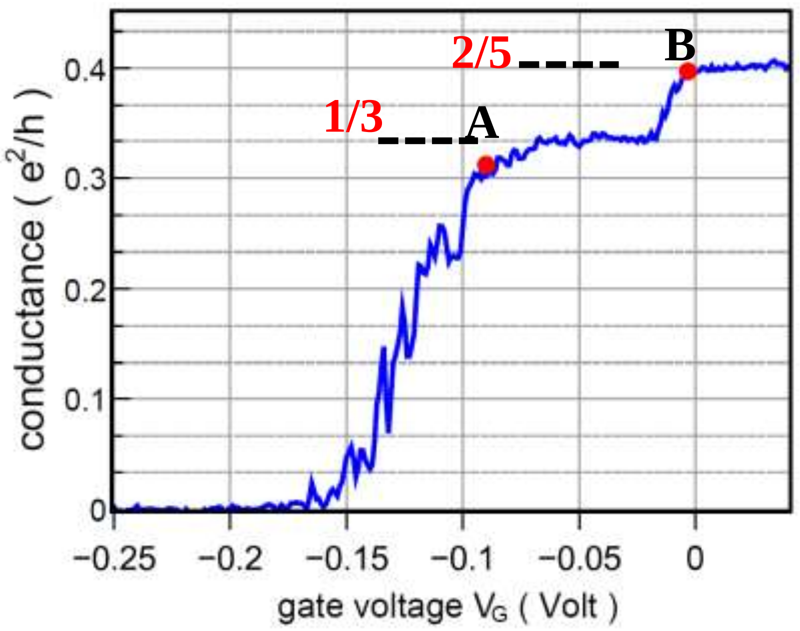



2.2 MHz
Δf=~150kHz

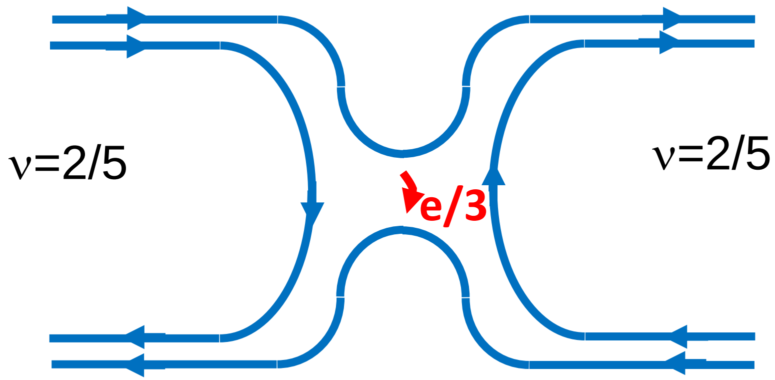
Helium-free Cryoconcept® crysostat

**14 Tesla** Dry Magnet
**13mK** base temperature

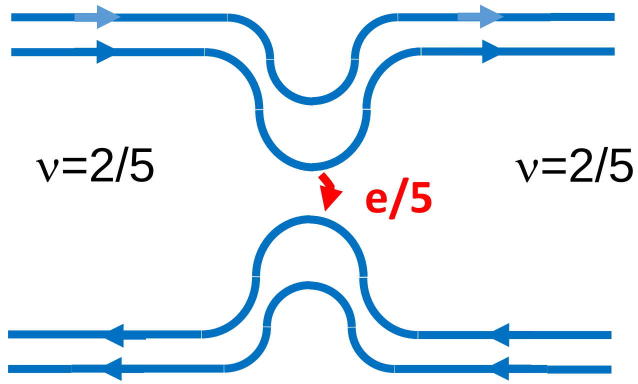

microwave
Photons
hf
B=11.2 Tesla
to cryo-amp
(1)
V_G
(0)
e*=e/5
ν_B=2/5
V_dc
to cryo-amp
(2)
V_G


2/5
B
1/3
A
0.4
0.3
0.2
0.1
0
conductance ( e^2/h )
−0.25
−0.2
−0.15
−0.1
−0.05
0
gate voltage V_G ( Volt )

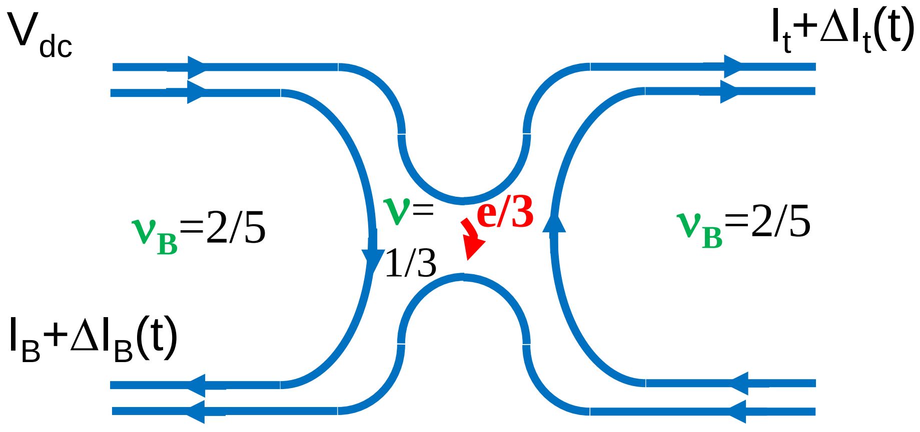
case **A**:

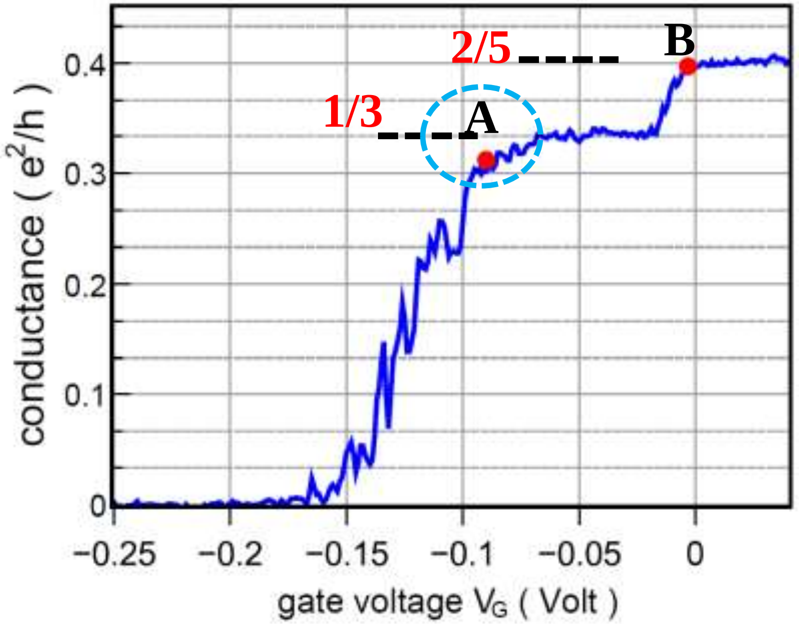

ν=2/5
e/3
ν=2/5

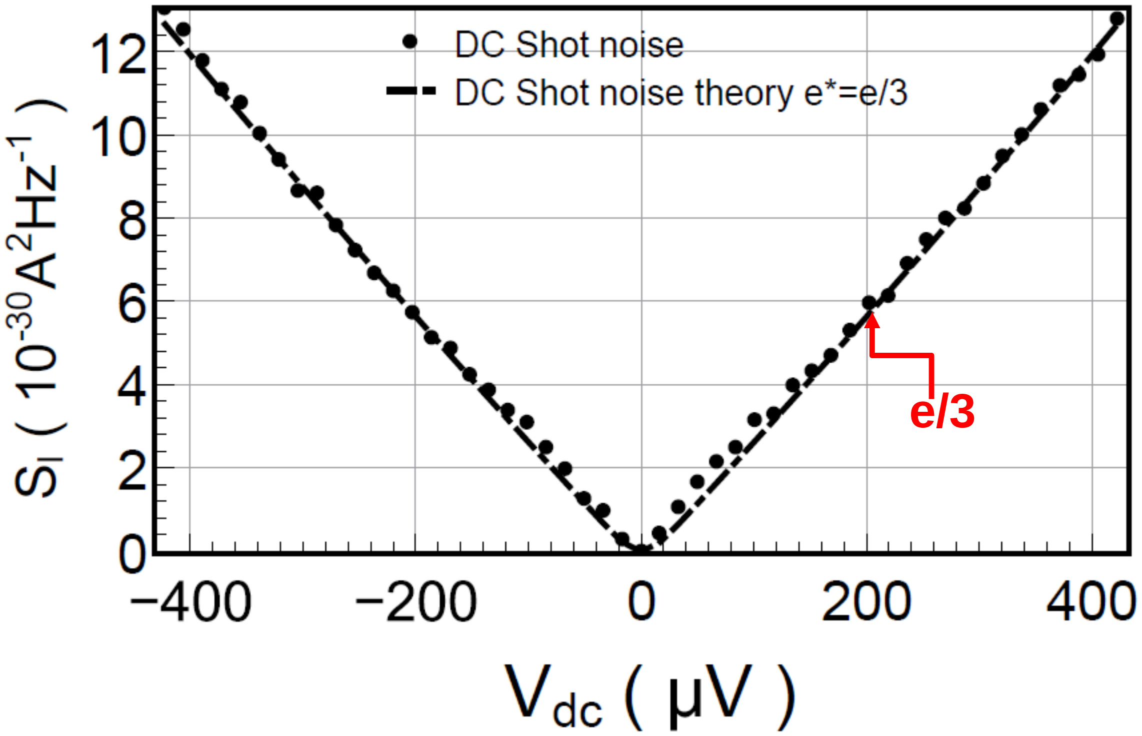
case **B**:


ν=2/5
e/5
ν=2/5

# DC Shot noise for the 1/3-FQHE state



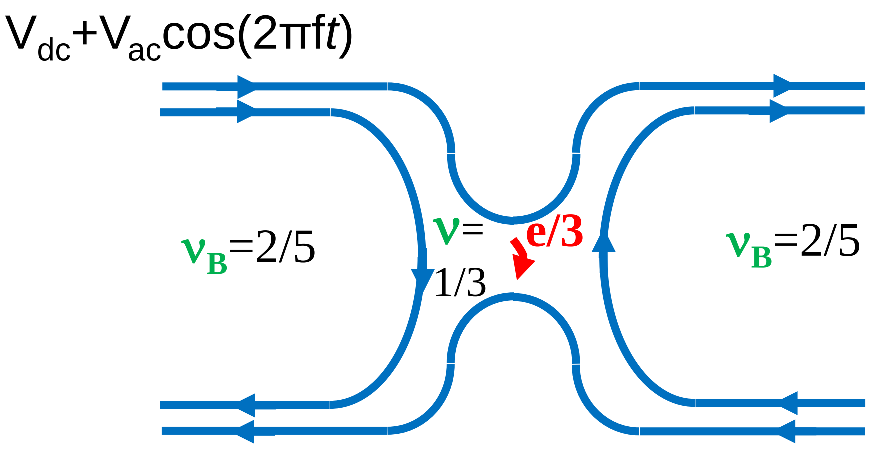

$$S_I^{DC} = 2e^* I_B \left[ \coth\left( \frac{e^* V_{dc}}{2k_B T} \right) - \frac{2k_B T}{e^* V_{dc}} \right]$$

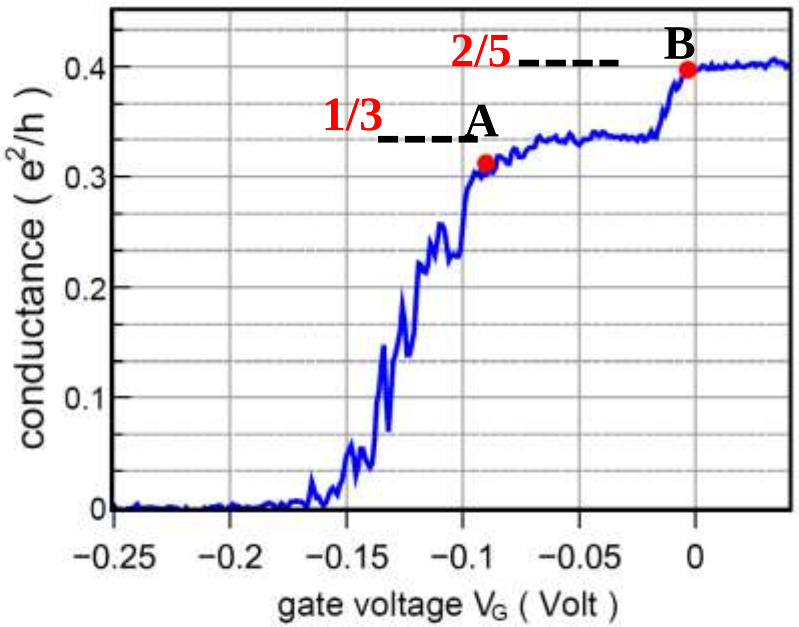
$e^* = e/3$ !

confirms '97-'98 experiments

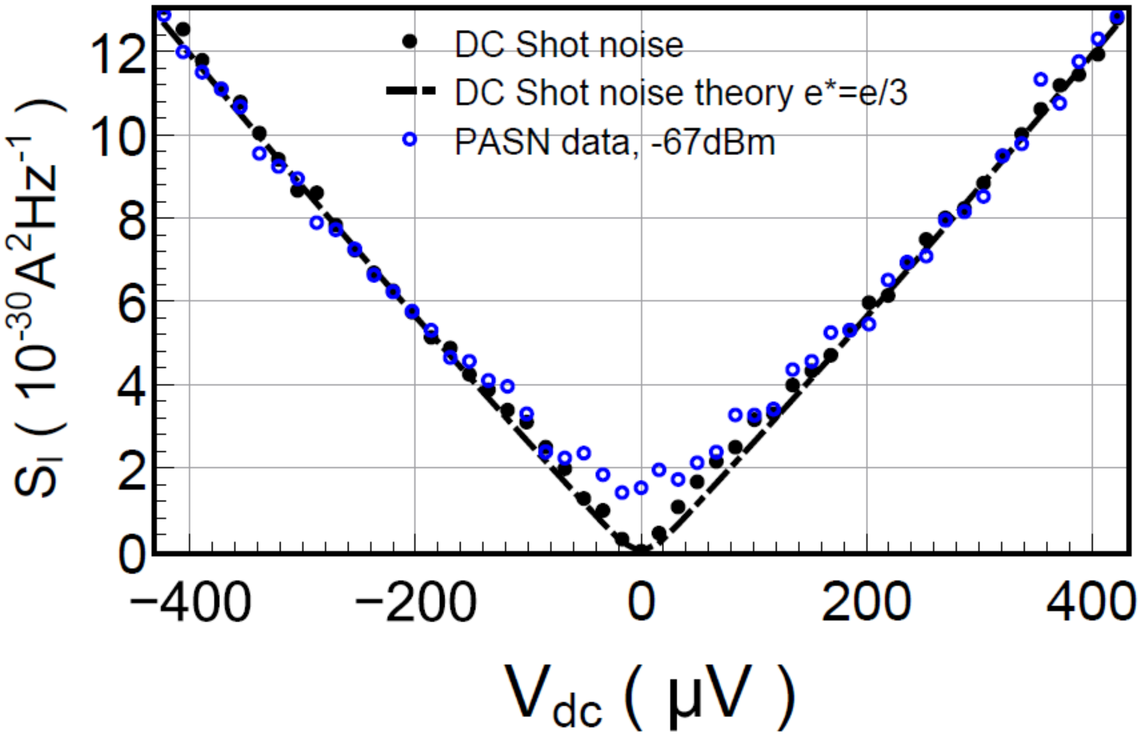
(Saclay PRL 97, Weizmann Nat. 97 and 99, NTT 2015)

# Photon-Assisted Shot Noise for the 1/3-FQHE state



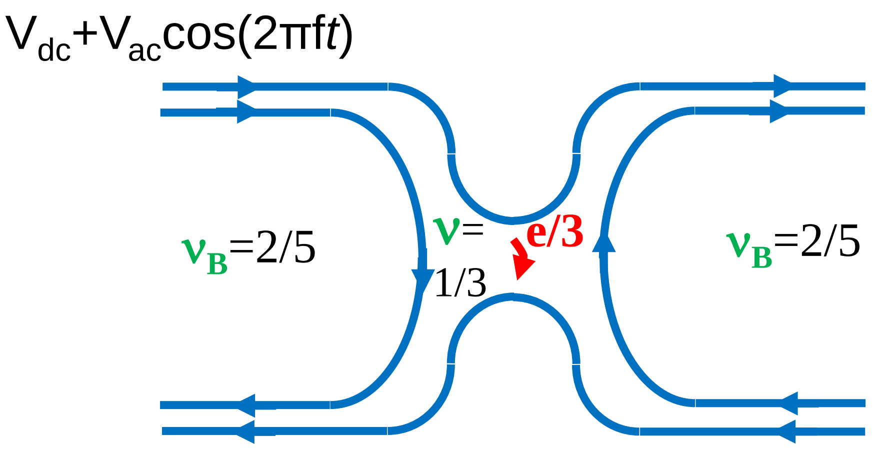

*f*=22GHz

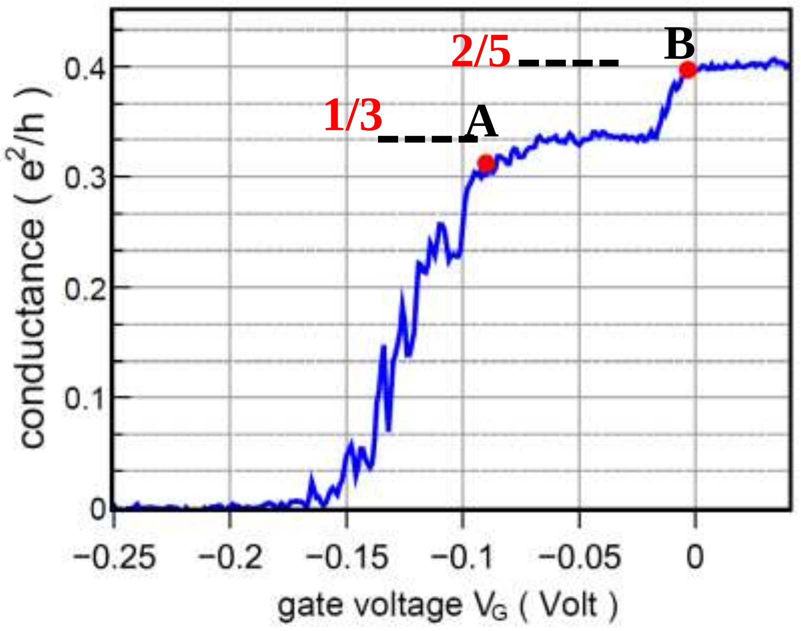



$$V(t)=V_{dc}+V_{ac}\cos(2\pi ft)$$

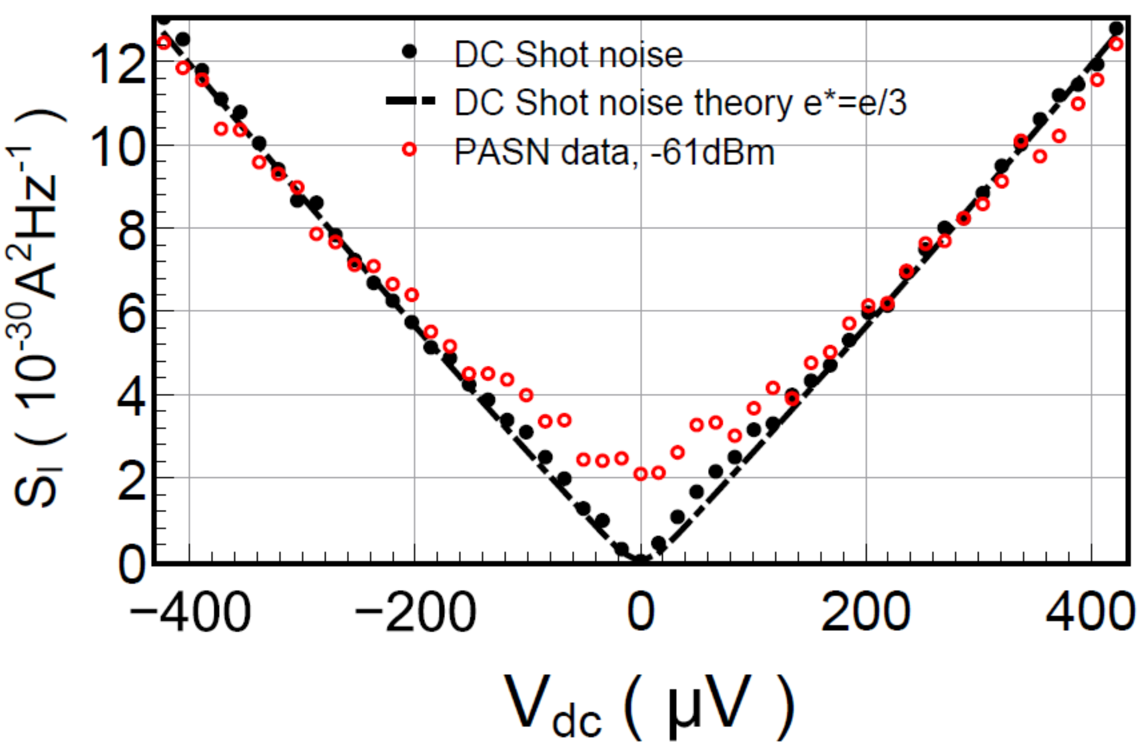
$$V_{ac}\approx 100\ \mu\text{V} \text{ for } -67\text{dBm}$$

# Photon-Assisted Shot Noise for the 1/3-FQHE state




*f*=22GHz




$$V(t) = V_{dc} + V_{ac}\cos(2\pi f t)$$

$V_{ac} \approx 200\ \mu V$ for -61dBm

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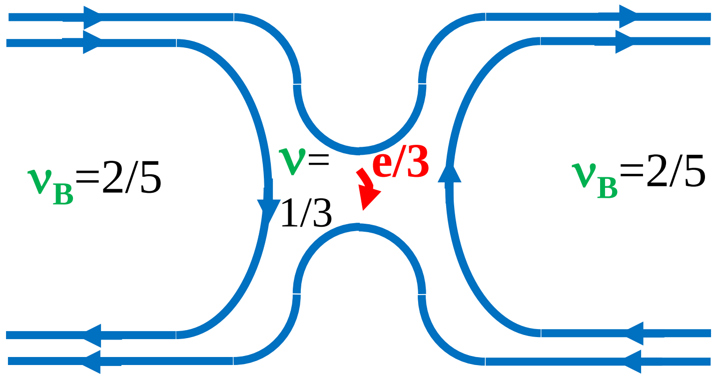


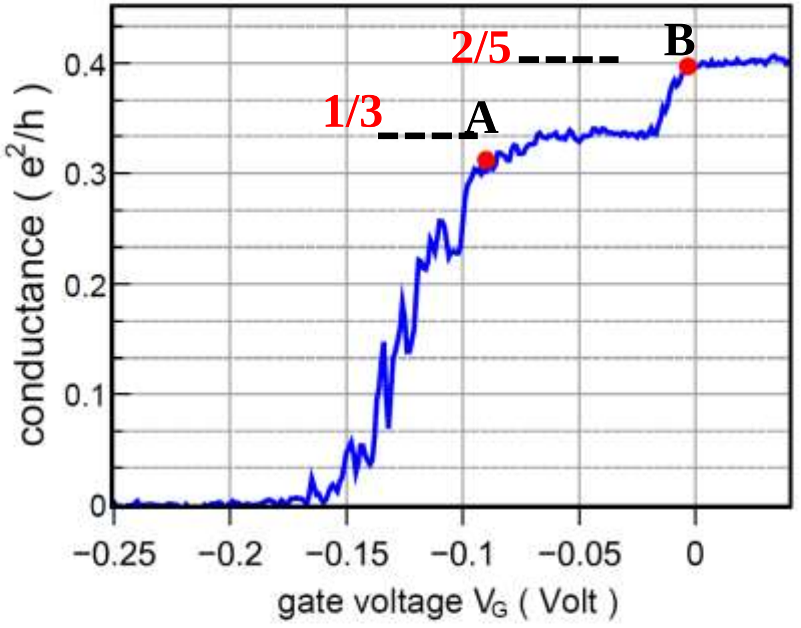


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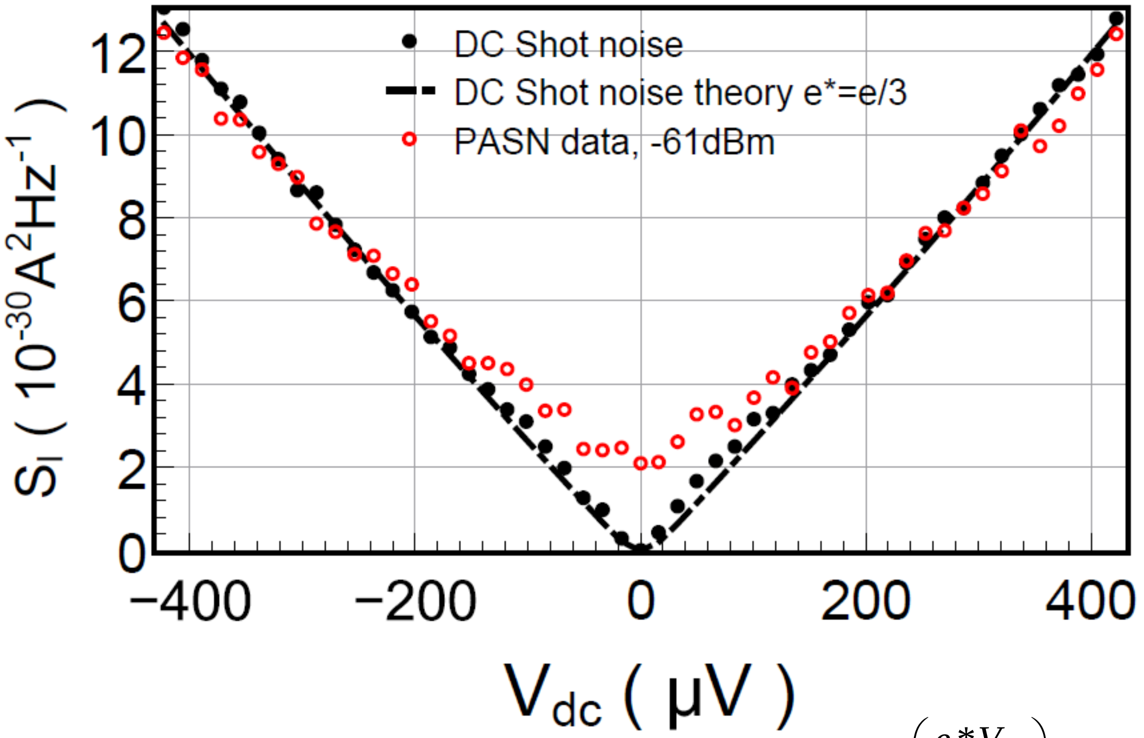



$$V(t)=V_{dc}+V_{ac}\cos(2\pi f t)$$

$$p_0=J_0\left(\frac{e^*V_{ac}}{hf}\right)$$

$V_{ac}\approx 200\ \mu V$ for -61dBm

$$p_1=-p_{-1}=J_1\left(\frac{e^*V_{ac}}{hf}\right)$$

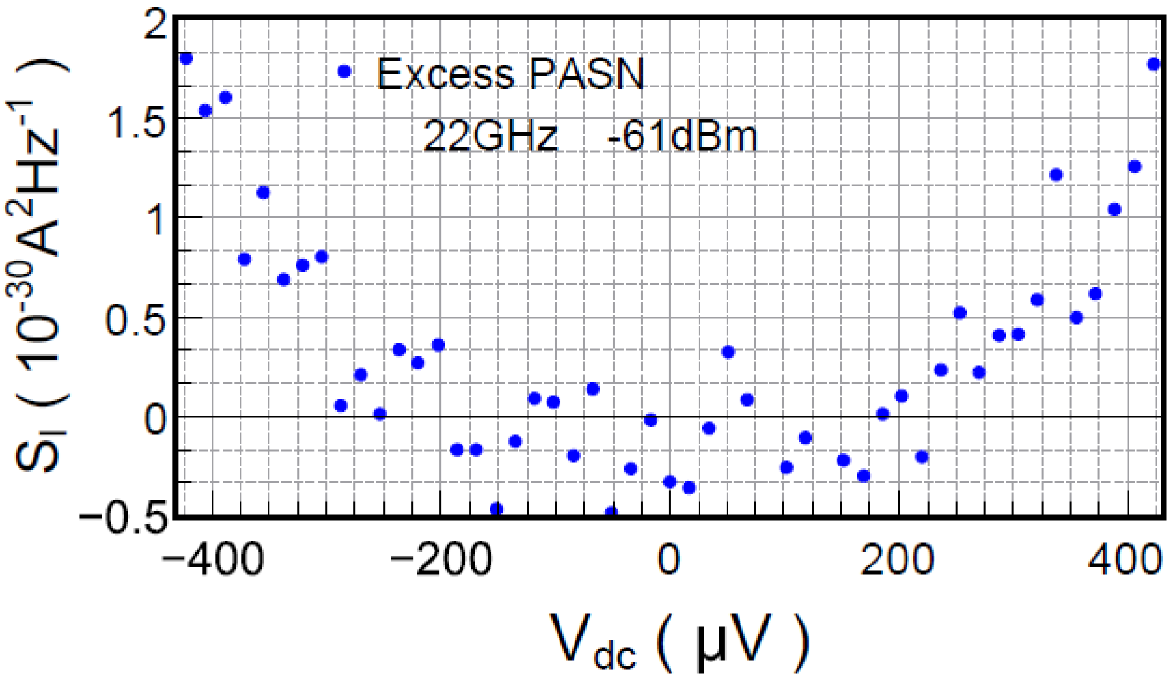
$$S_I^{PASN}(V_{dc})\,?=?\,|p_0|^2 S_I^{DC}(V_{dc})+|p_1|^2\left[S_I^{DC}(V_{dc}-hf/e^*)+S_I^{DC}(V_{dc}+hf/e^*)\right]$$

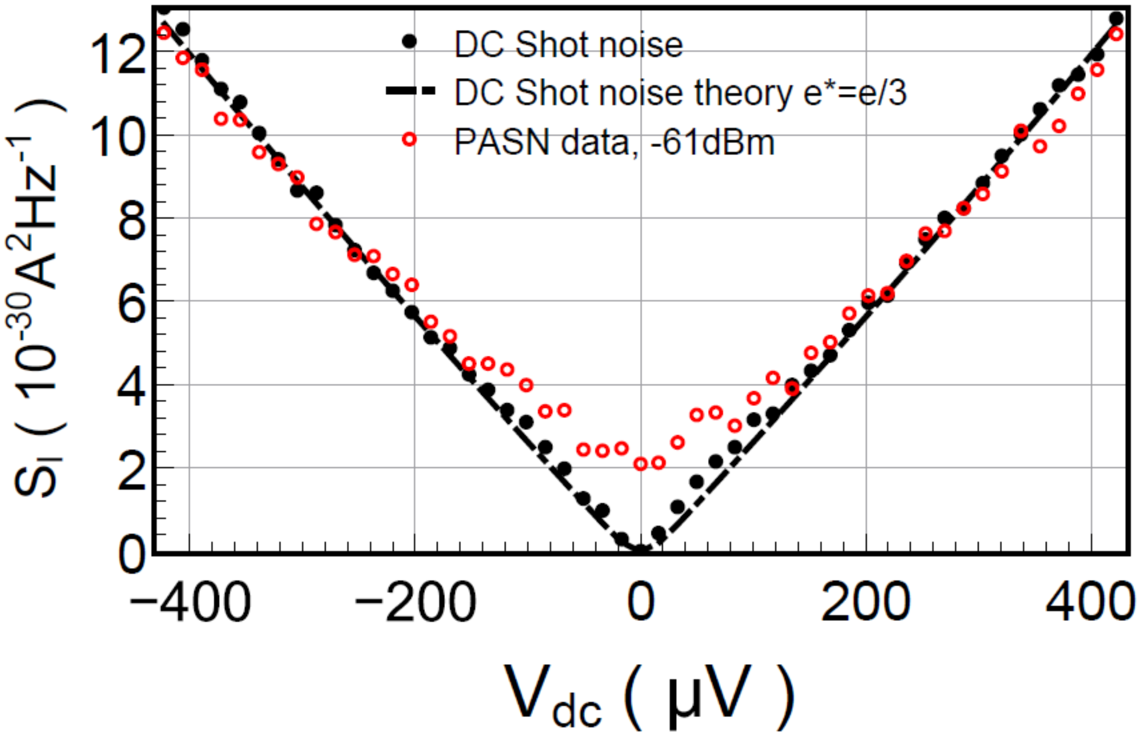
# Excess PASN for the 1/3-FQHE state

## Killing the non photon-assisted part !

Excess PASN:

$$\Delta S_I = S_I^{PASN}(V_{dc}) - |p_0|^2 S_I^{DC}(V_{dc})$$
$$= |p_1|^2 \left[ S_I^{DC}(V_{dc} - hf/e^*) + S_I^{DC}(V_{dc} + hf/e^*) \right]$$

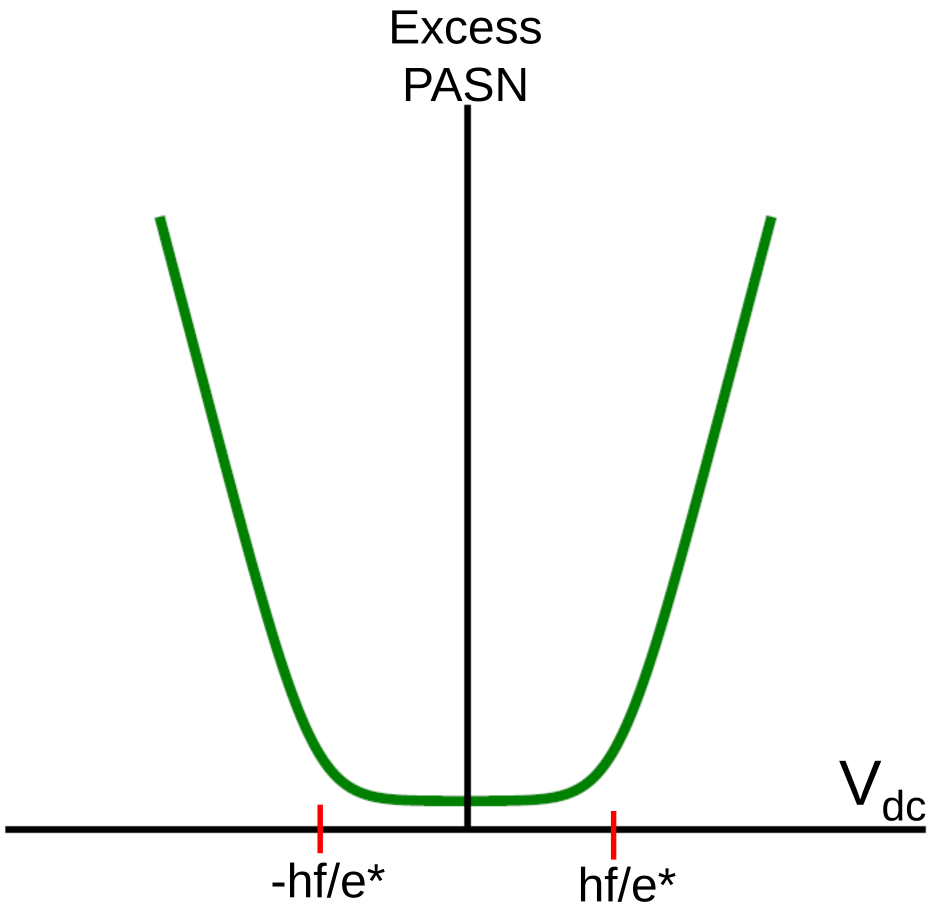


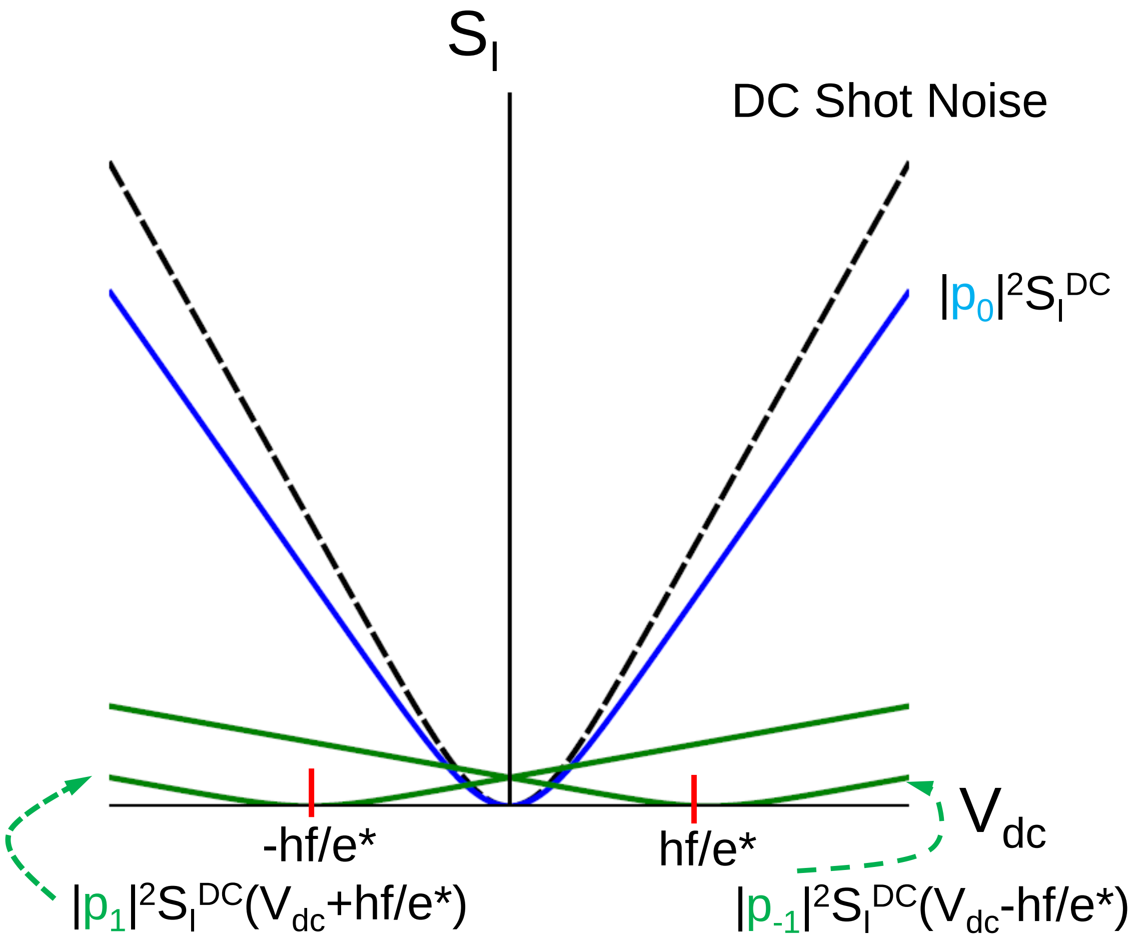




Finding *a flat variation* for the low $|V_{dc}|$ range provides a determination of $|p_0|^2$

# Excess PASN for the 1/3-FQHE state

WHY a FLAT VARIATION?



# Excess PASN for the 1/3-FQHE state

## Killing the non photon-assisted part !

Excess PASN:

$$\Delta S_I = S_I^{PASN}(V_{dc}) - |p_0|^2 S_I^{DC}(V_{dc})$$
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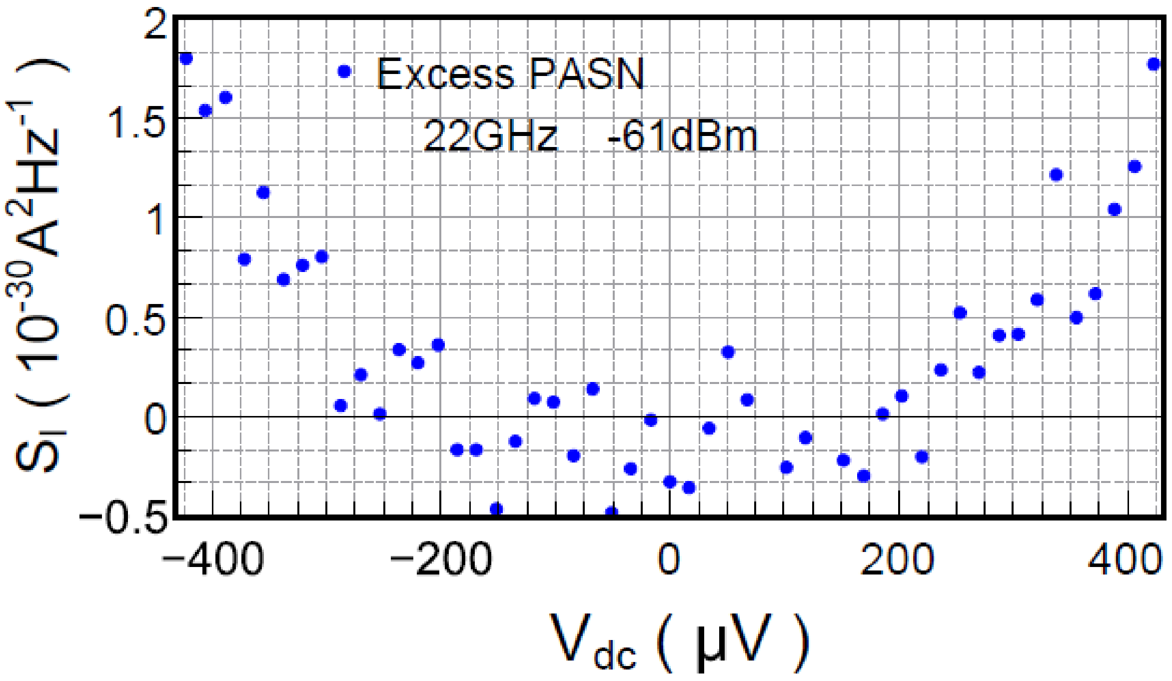


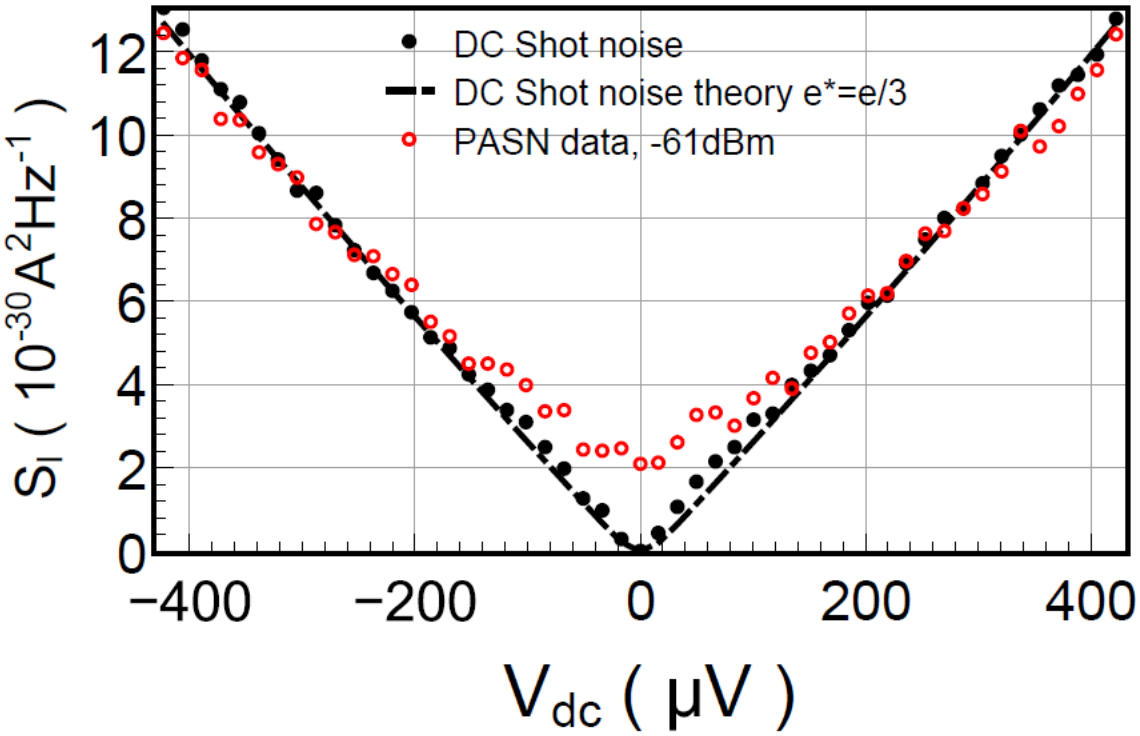


Finding a flat variation for the low $|V_{dc}|$ range provides a determination of $|p_0|^2$

as $|p_0|^2 + 2|p_1|^2 \approx 1$ this gives $|p_1|^2$

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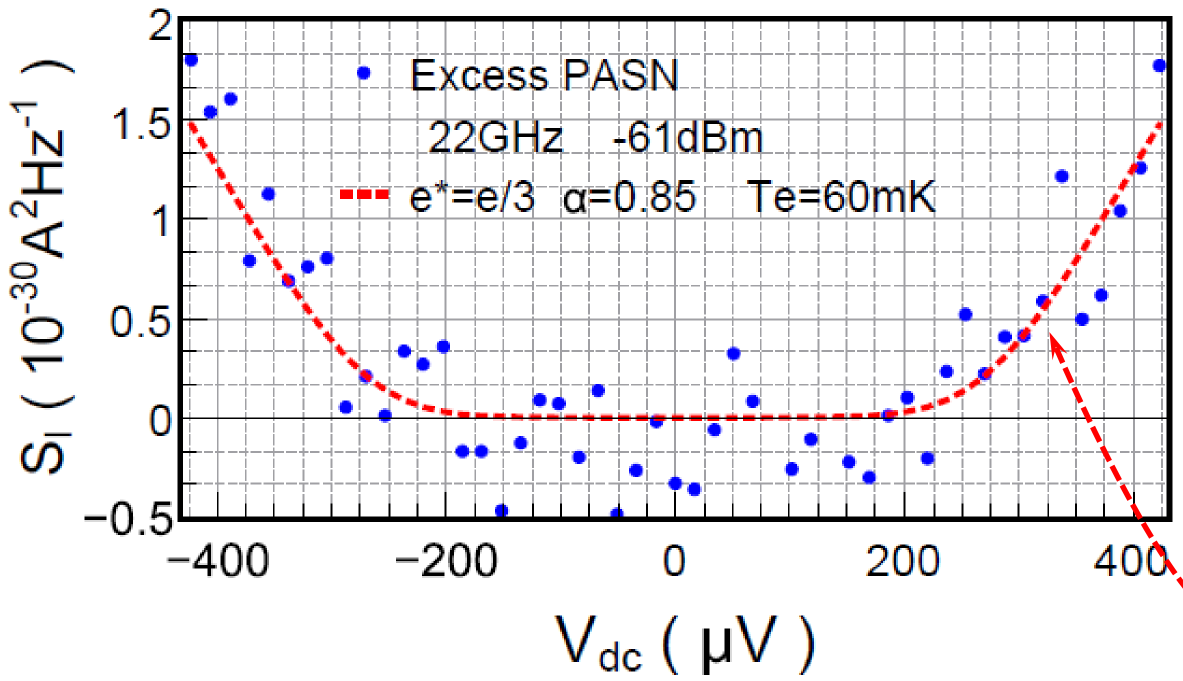


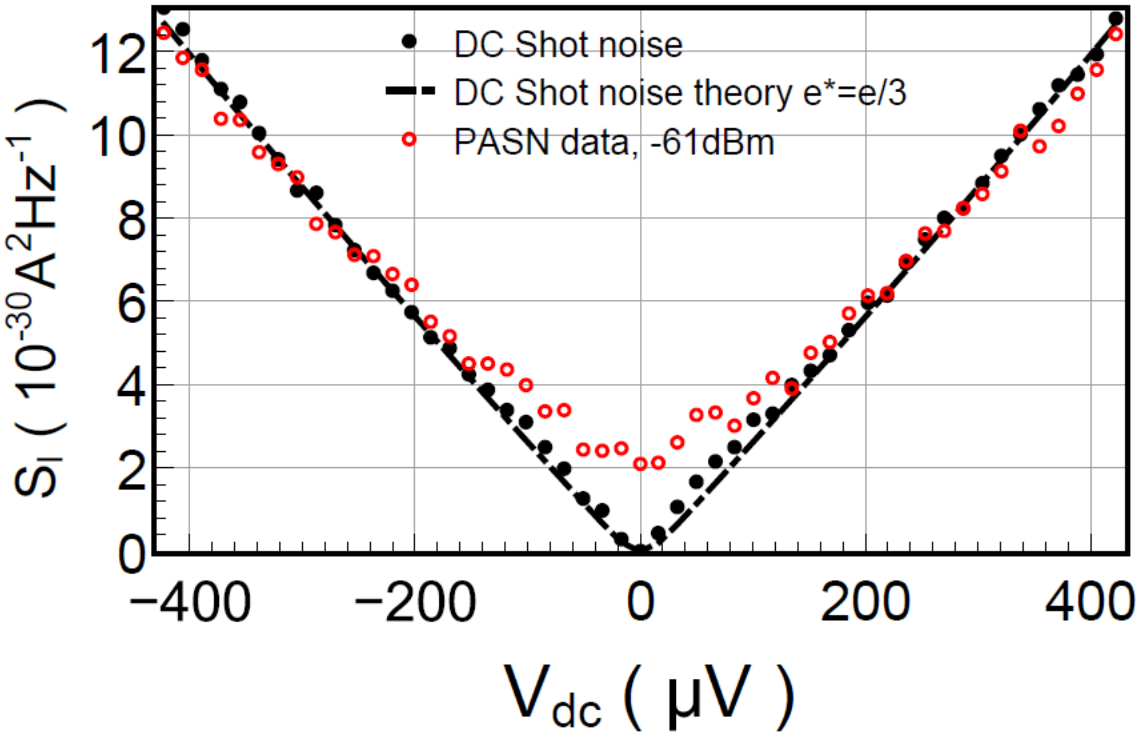


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comparison using $f_{Josephson} = e^* V_{dc}/h$ with $e^*=e/3$

# Josephson relation for the 1/3-FQHE state

CHECKING the FREQUENCY DEPENDENCE of Excess PASN:

$$\Delta S_I = S_I^{PASN}(V_{dc}) - |p_0|^2 S_I^{DC}(V_{dc})$$
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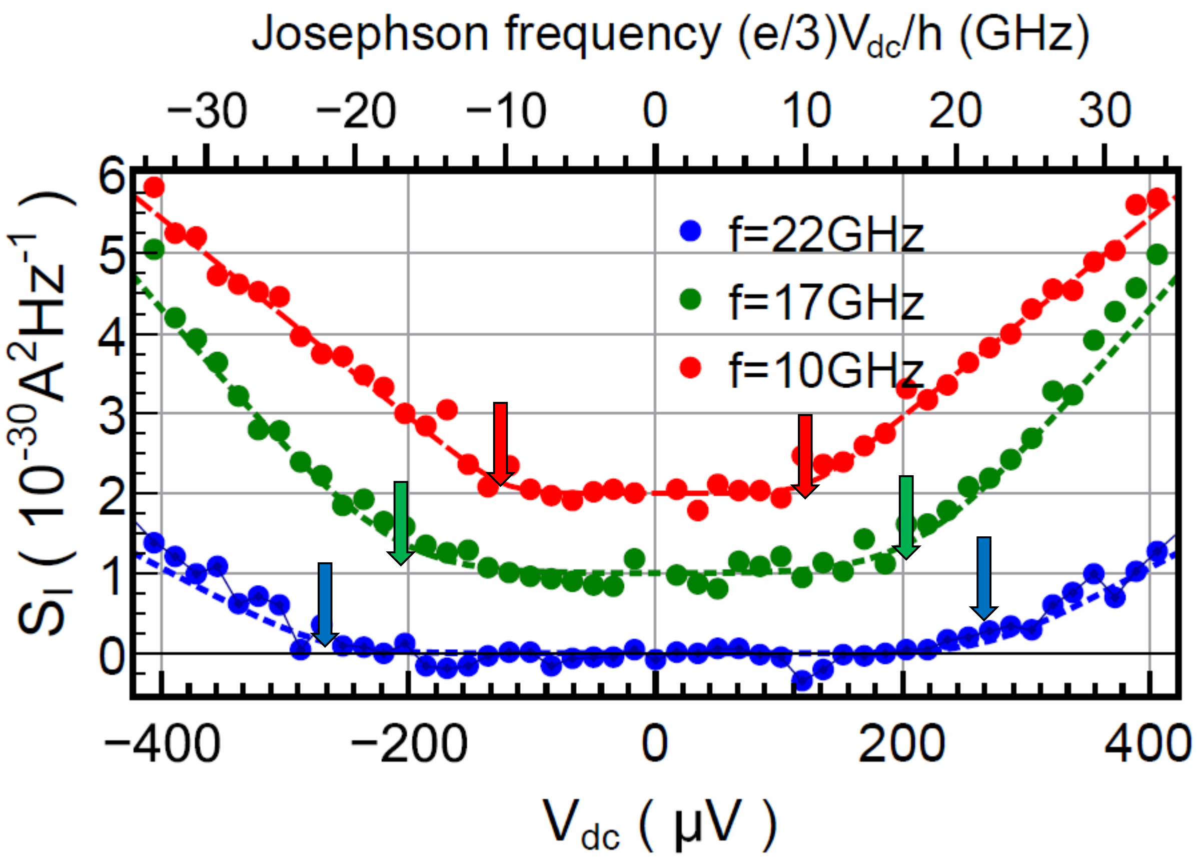
threshold voltage : $V_J = hf/e^*$ scales with frequency!



# New Measurement of e* for the 1/3-FQHE State

MEASURING e* from Excess PASN:

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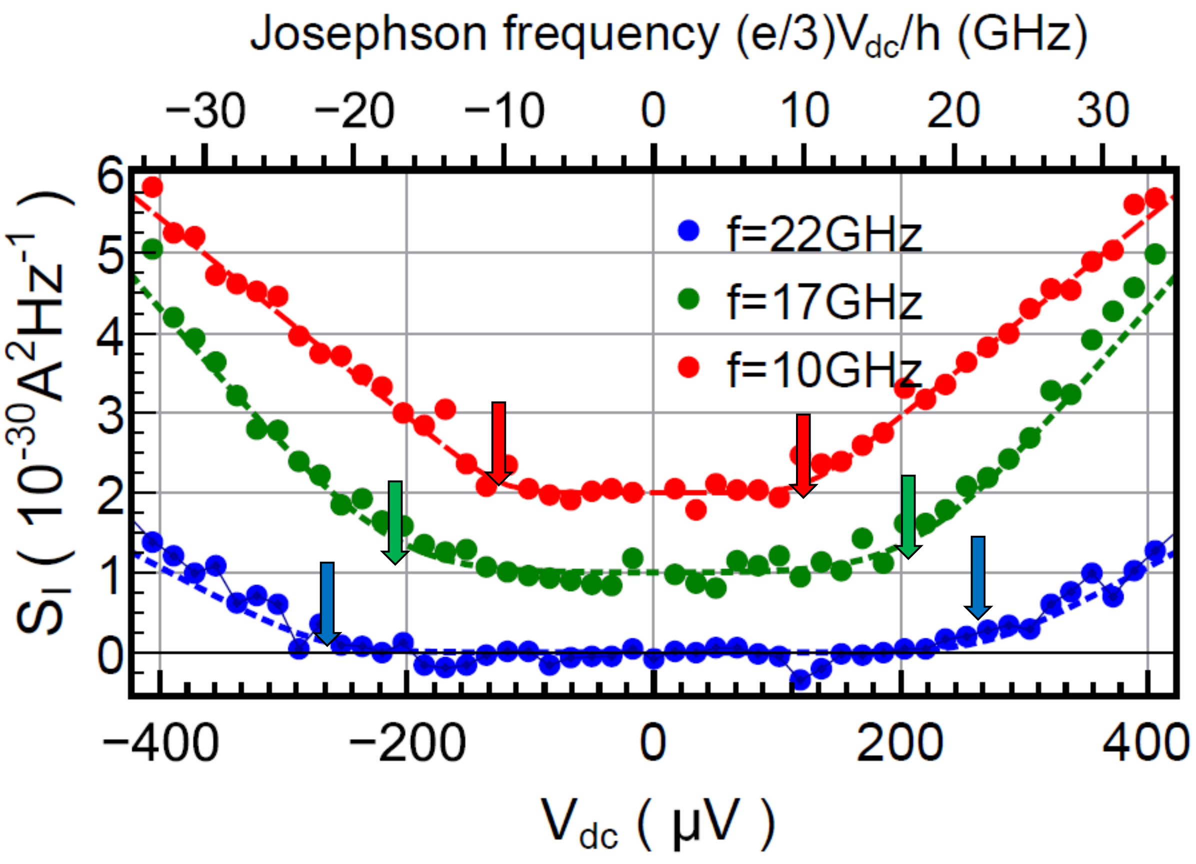
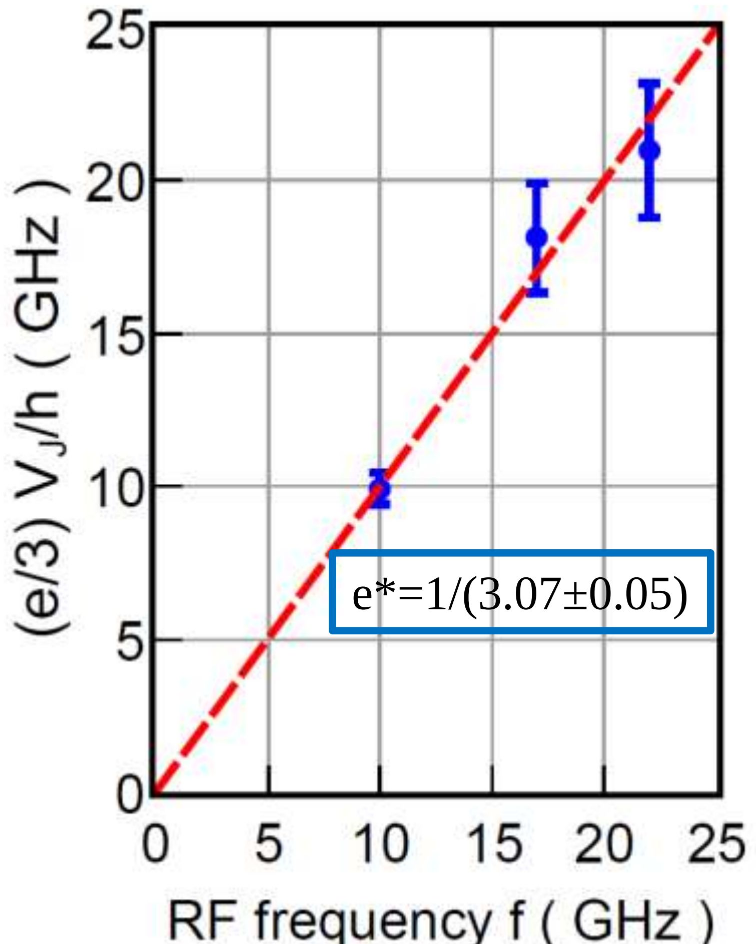
Best fit of data with e* free parameter



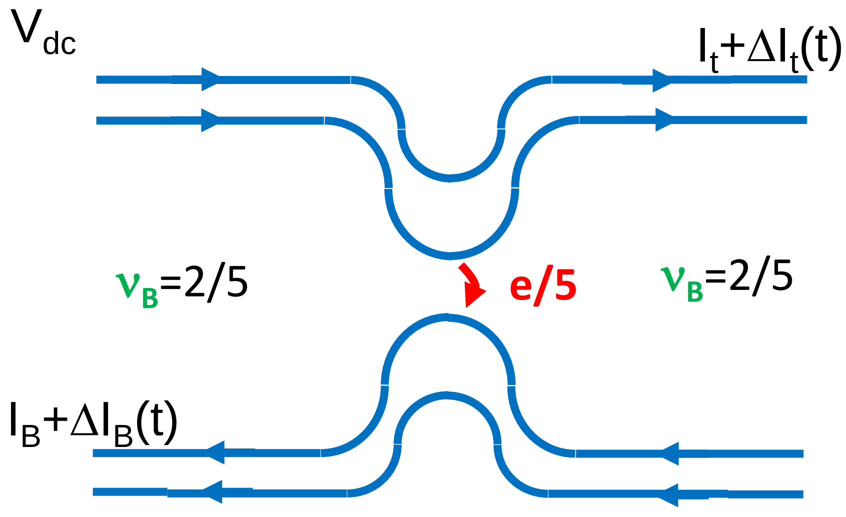
# OUTLINE

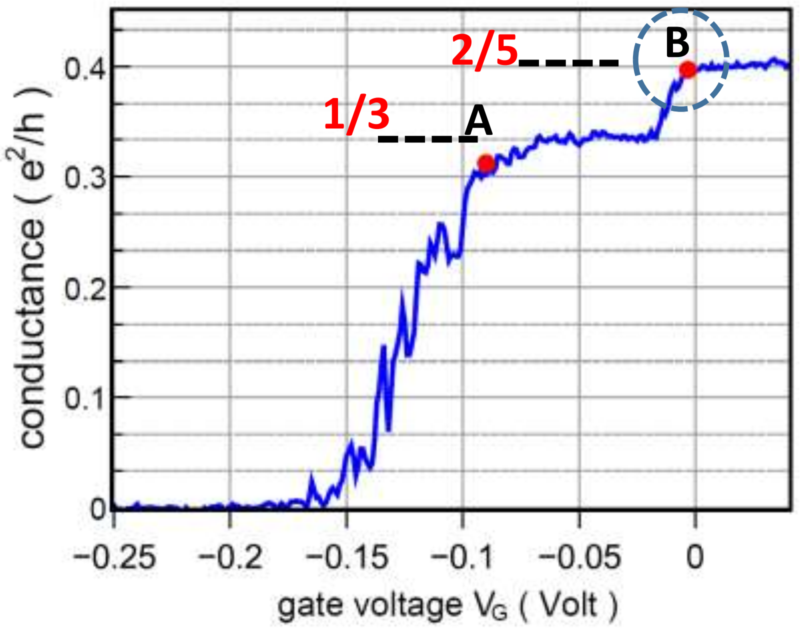
- Quantum Hall edge states and Fractional Quantum Hall Effect
- PHOTON-ASSISTED TRANSPORT
  - Photon-assisted processes
  - A JOSEPHSON Relation for Photon Assisted Shot Noise (PASN)
- Experimental Results
  - e*=e/3
  - e*=e/5
- CONCLUSION and PERSPECTIVES

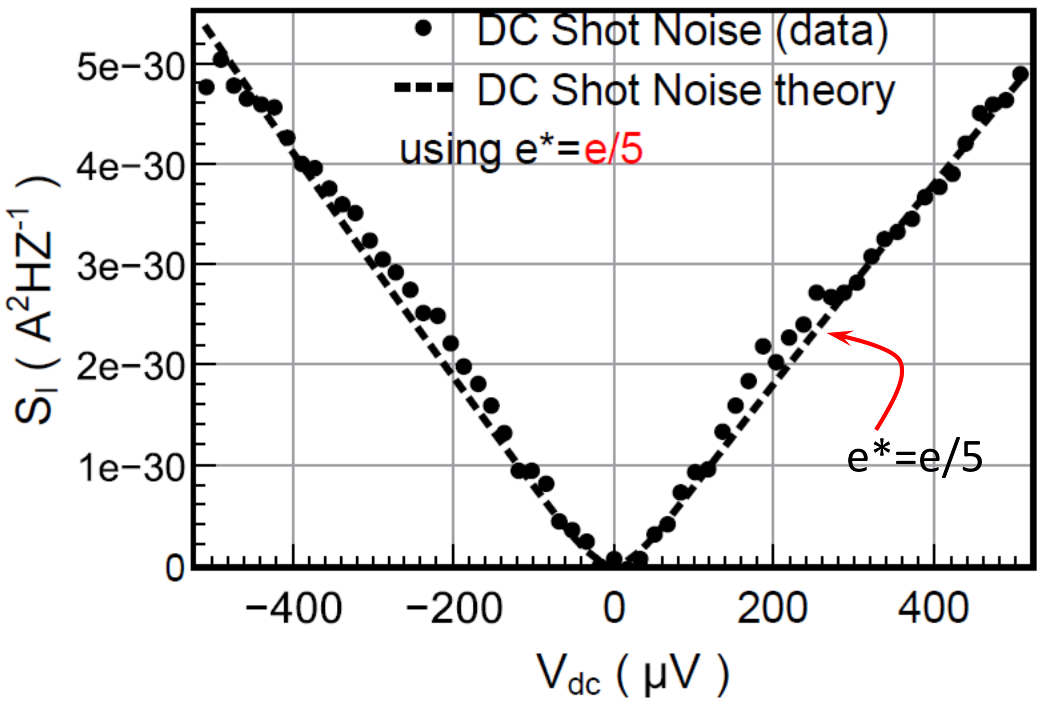
$$f_{J.} = \frac{e^*V}{h}$$

X. G. Wen (1991)

# DC Shot noise for the 2/5-FQHE state



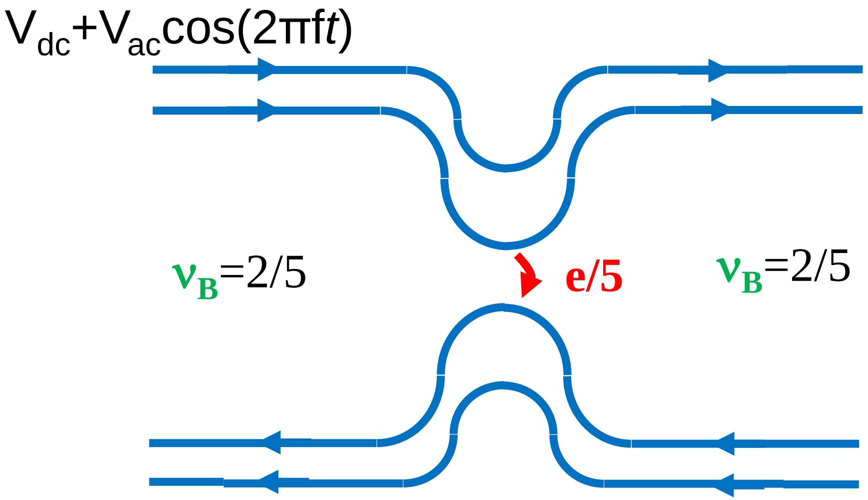



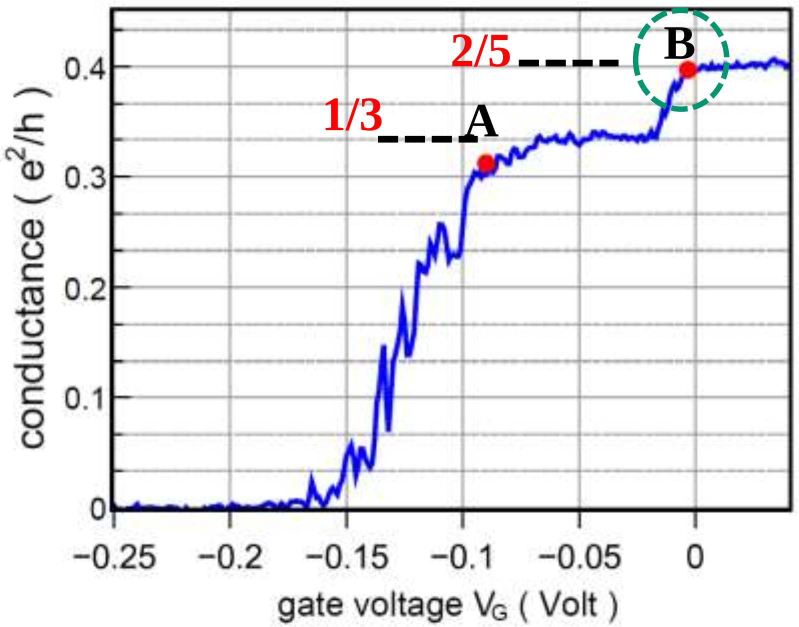

$$S_I^{DC} = 2e^* I_B \left[ \coth\left( \frac{e^* V_{dc}}{2k_B T} \right) - \frac{2k_B T}{e^* V_{dc}} \right] \propto -\langle \Delta I_B \Delta I_t \rangle$$

e*= e/5 !

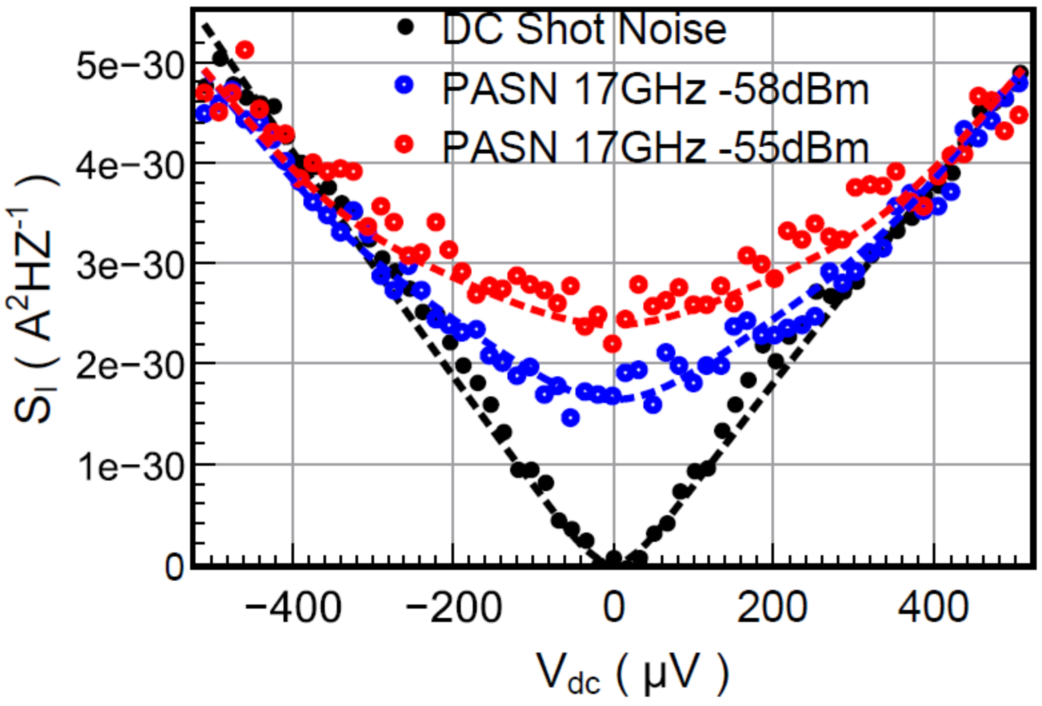
confirms Weizmann results (Reznikov 1999) on 2/5

# Photon-Assisted Shot Noise for the 2/5-FQHE state




*f*=17GHz




$$V(t) = V_{dc} + V_{ac}\cos(2\pi f t)$$

$V_{ac} \approx 300\ \mu\text{V}$ for -58dBm

$\approx 400\ \mu\text{V}$ for -55dBm

# Excess PASN for the 2/5-FQHE state

## Killing again the non photon-assisted part !

Excess PASN:

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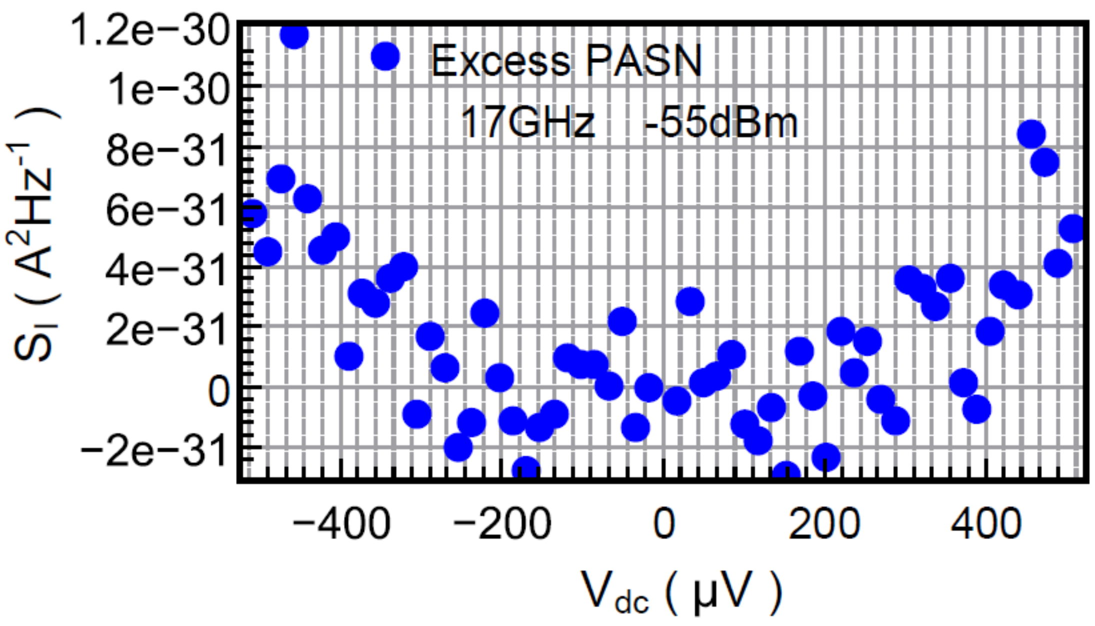


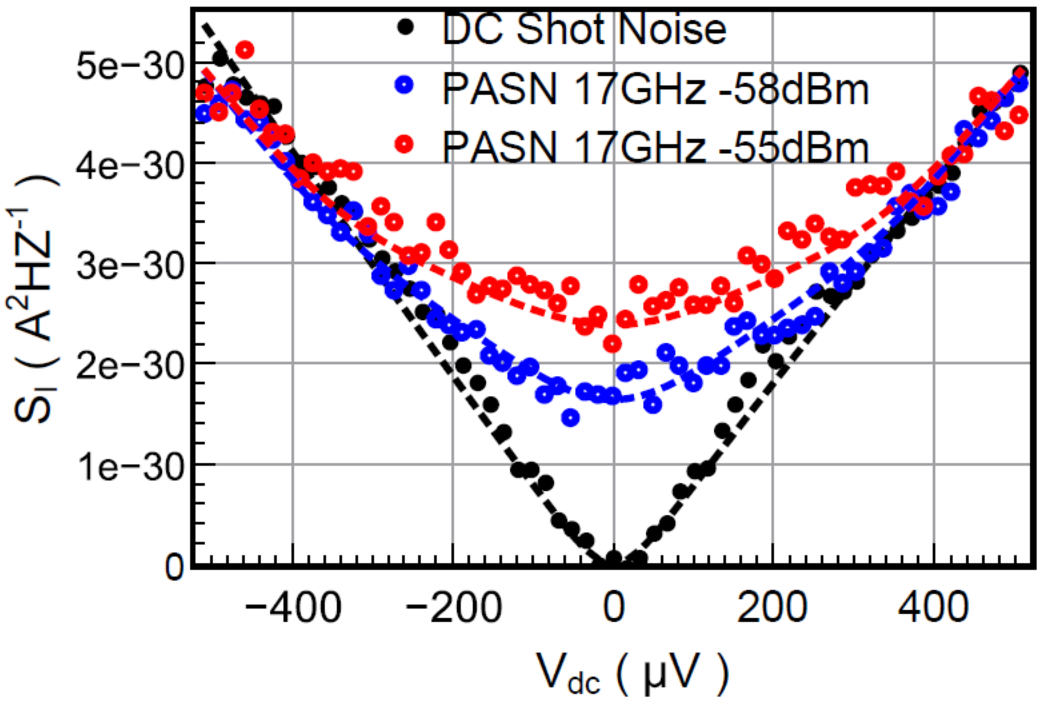


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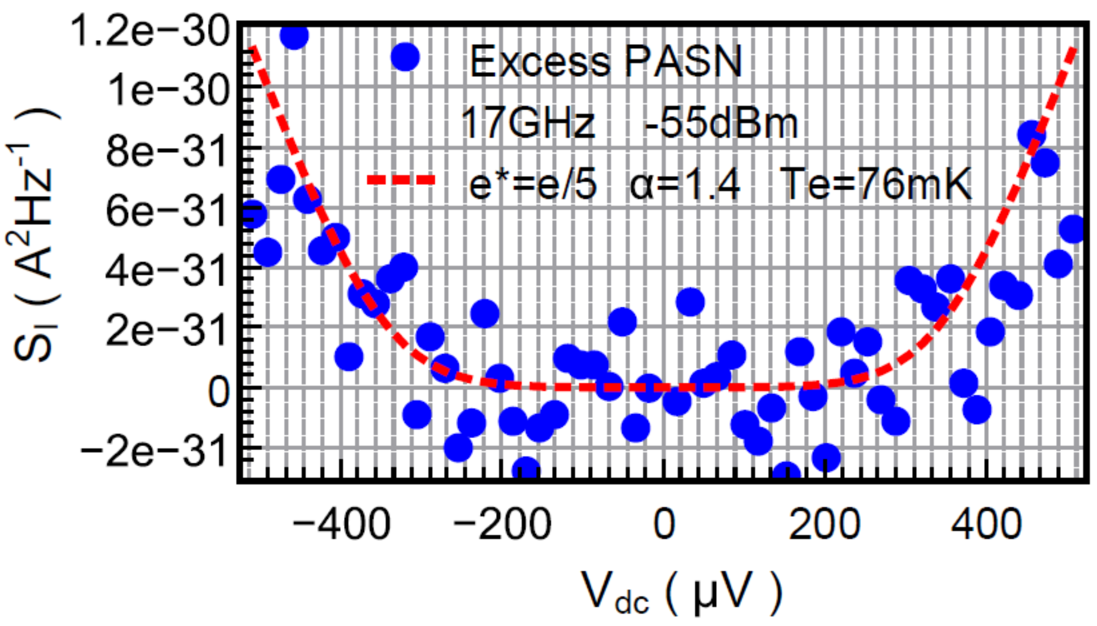


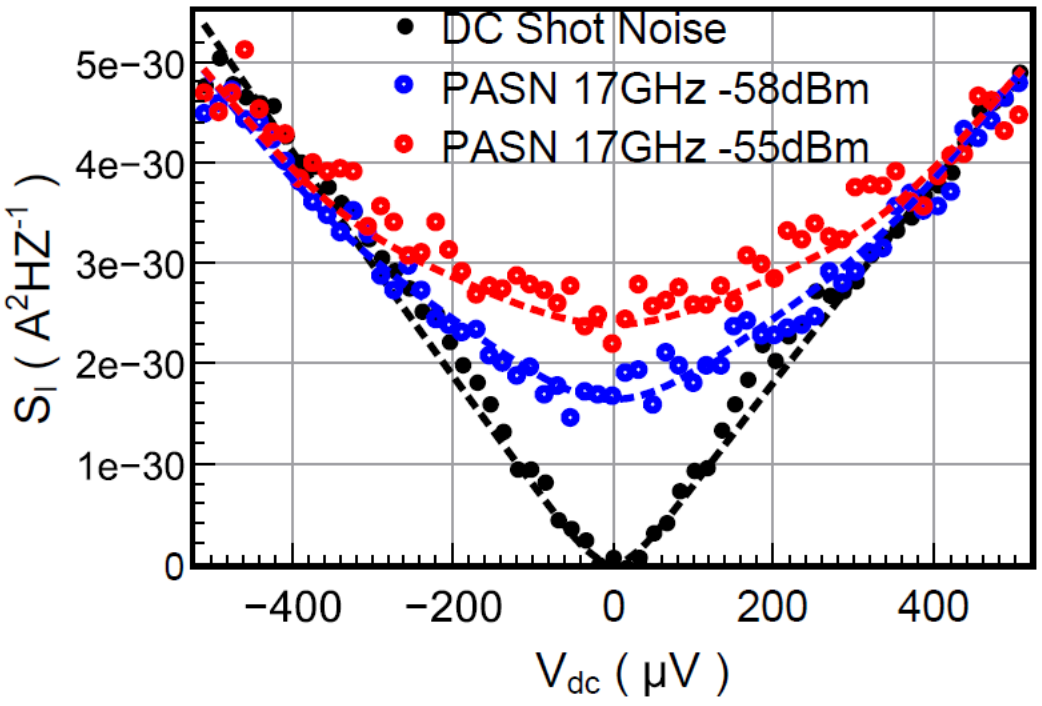


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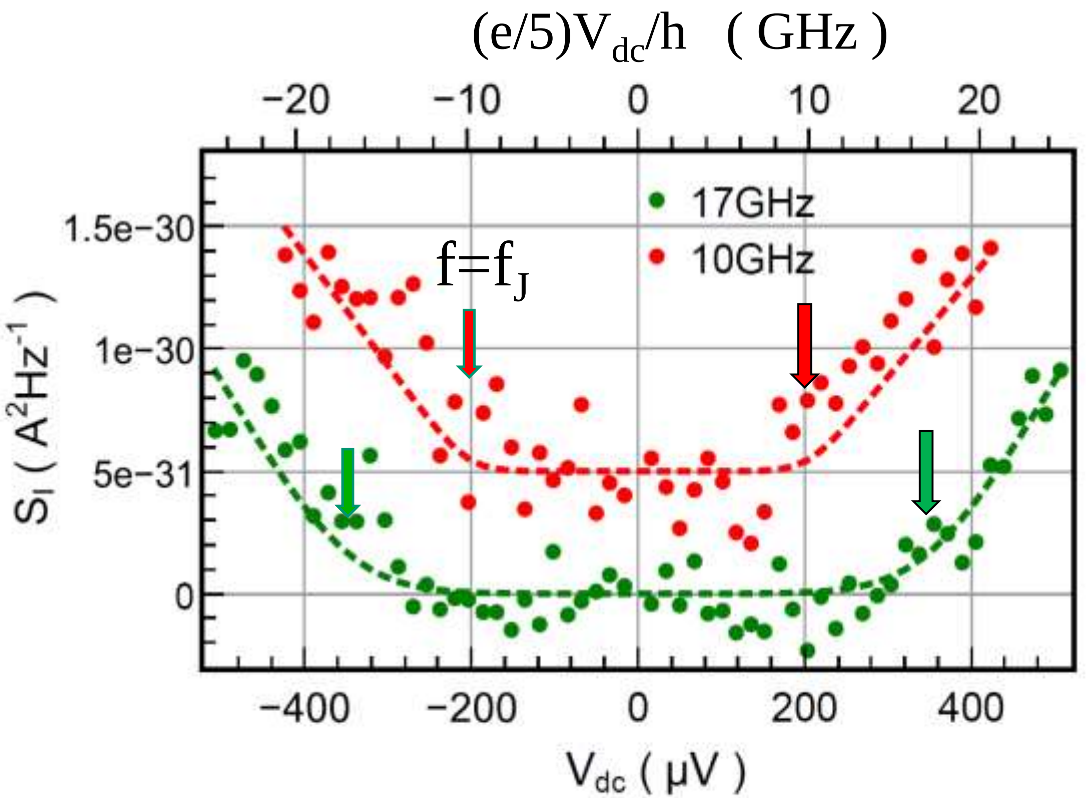

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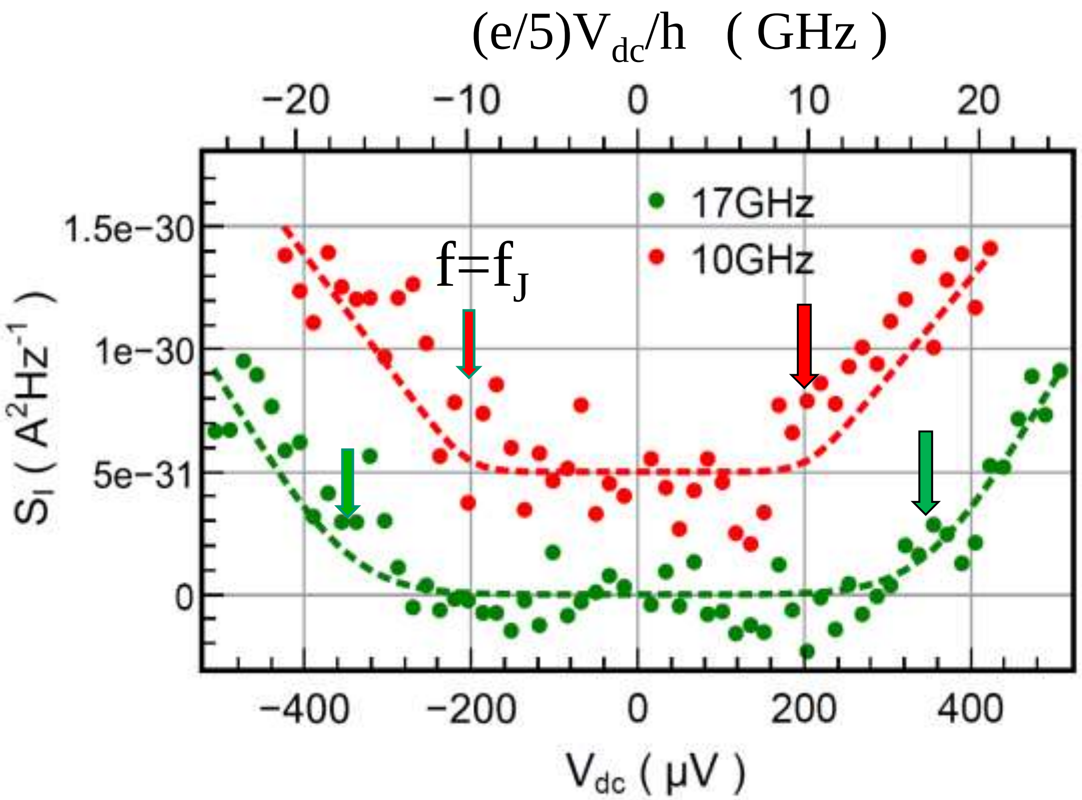
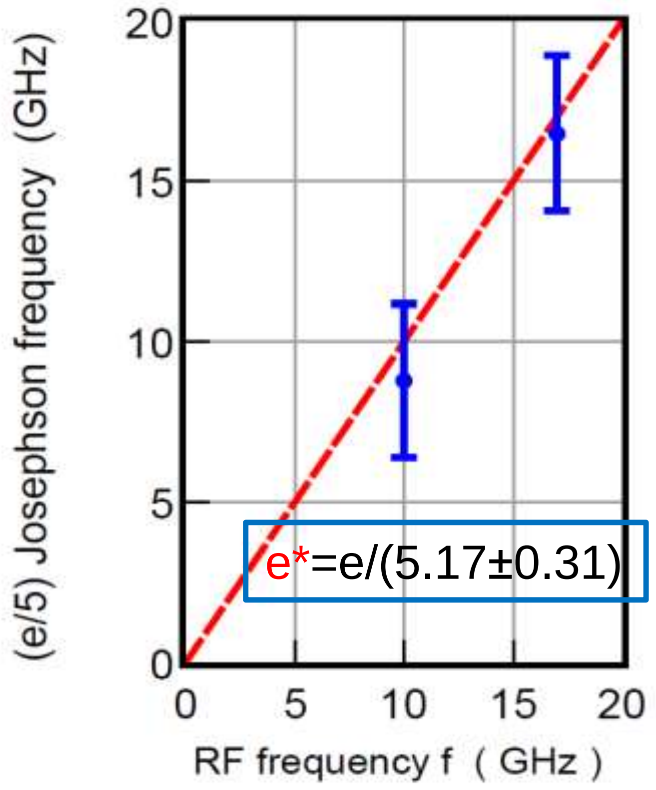
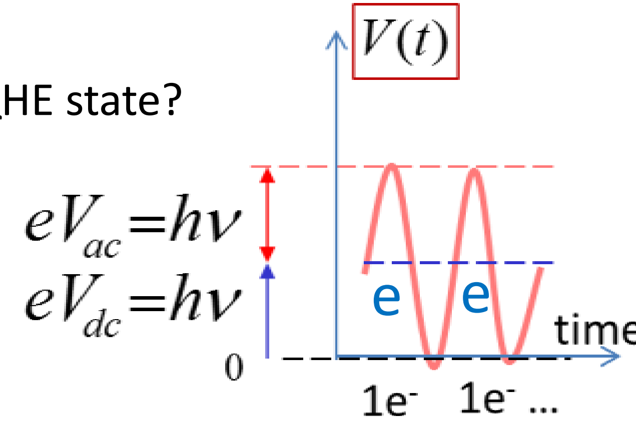


# CONCLUSION

- FQHE e*=e/3 and e/5 abelian *anyons can be manipulated* with microwave by well-defined photon-assisted processes. What about e/4 in non-abelian 5/2 FQHE state?
- Validates the possibility to realize on-demand single anyon sources for time domain *anyon braiding.*
- Based on Photon-Assisted Shot Noise (PASN)
- Shows evidence of the Josephson relation e*V/h=f predicted in 1991 by X.G. Wen*




Sine wave single charge pulses

( Old 1997 exp. )

**SCHOTTKY** (charge **granularity** )

$S_I = 2 e^* I_B$

weak signal but accurate

good signal but lack of accuracy, model dependent

( New 2019 exp. )

**PASN** Josephson Relation (photon **quantum** )

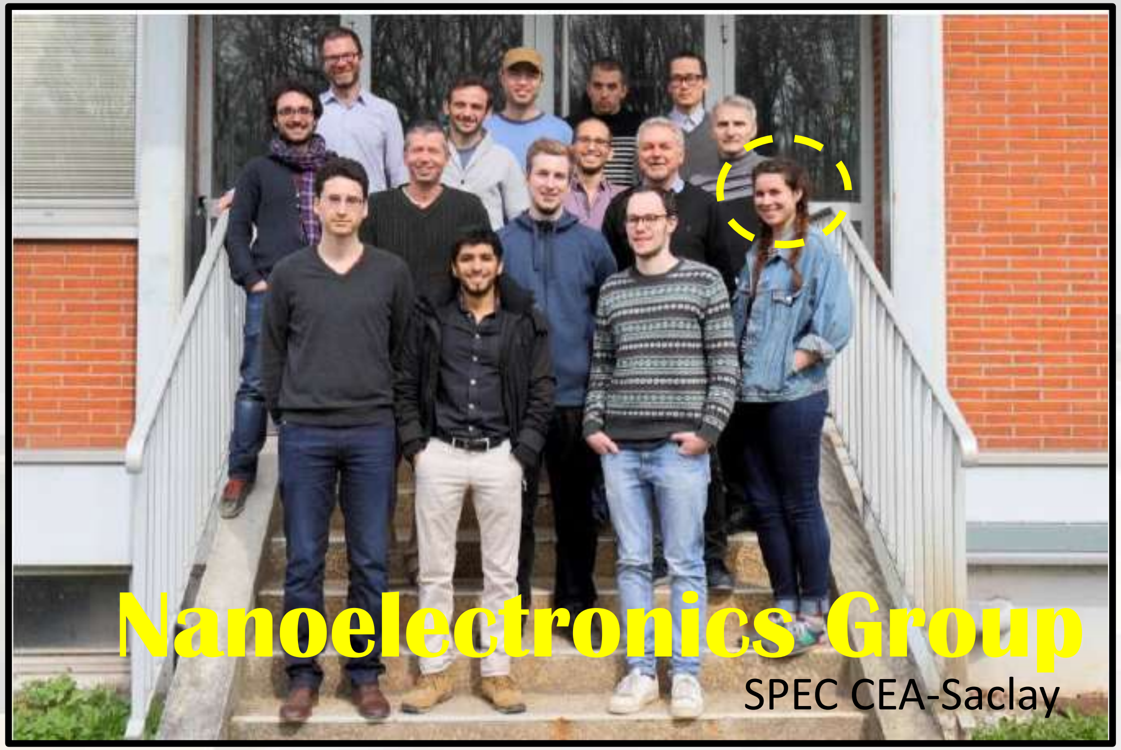
$h f = e^* V$

very accurate

good accuracy

*predicted for the current, see als
I. Safi +Sukhorukov (2010).

# ACKNOWLEDGEMENTS




OPEN POSITION
for 18-24 months
Post-doct.
(urgent)

X. Waintal
H. Saleur
I Safi
Th. Martin
M. Freedman
All members of Nanoelectronics Group at Saclay
The cryogeny Team

*ANR FullyQuantum AAP CE30*

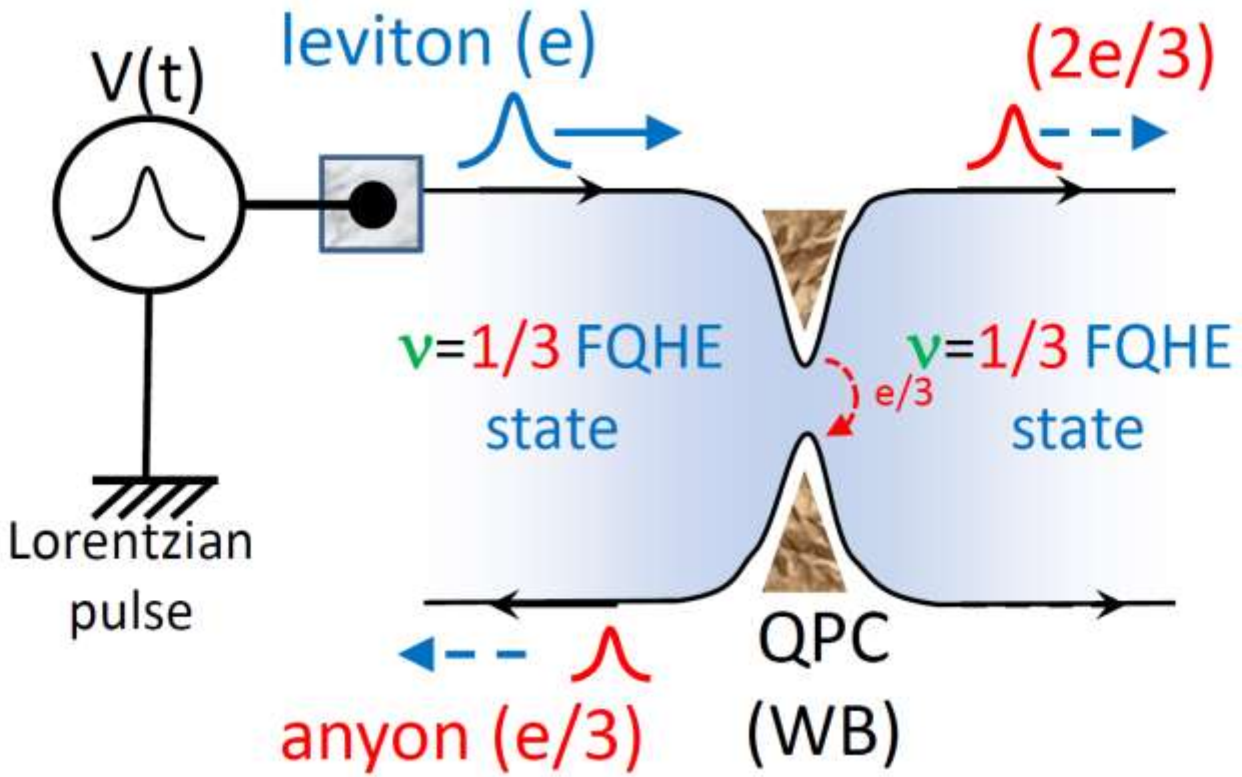
*The Josephson Frequency of fractionally charge anyons*
M. Kapfer, P. Roulleau, I. Farrer, D. A. Ritchie, and D. C. Glattli,
arXiv:1806.03117,
Published 24 January 2019 on ***Science***
**DOI: 10.1126/science.aau3539**

*Levitons :*
J. Dubois et al, Nature 502, 659 (2013*)*
T. Jullien et al., Nature 514, 603 (2014)

# PERSPECTIVE :
# ANYONS on DEMAND

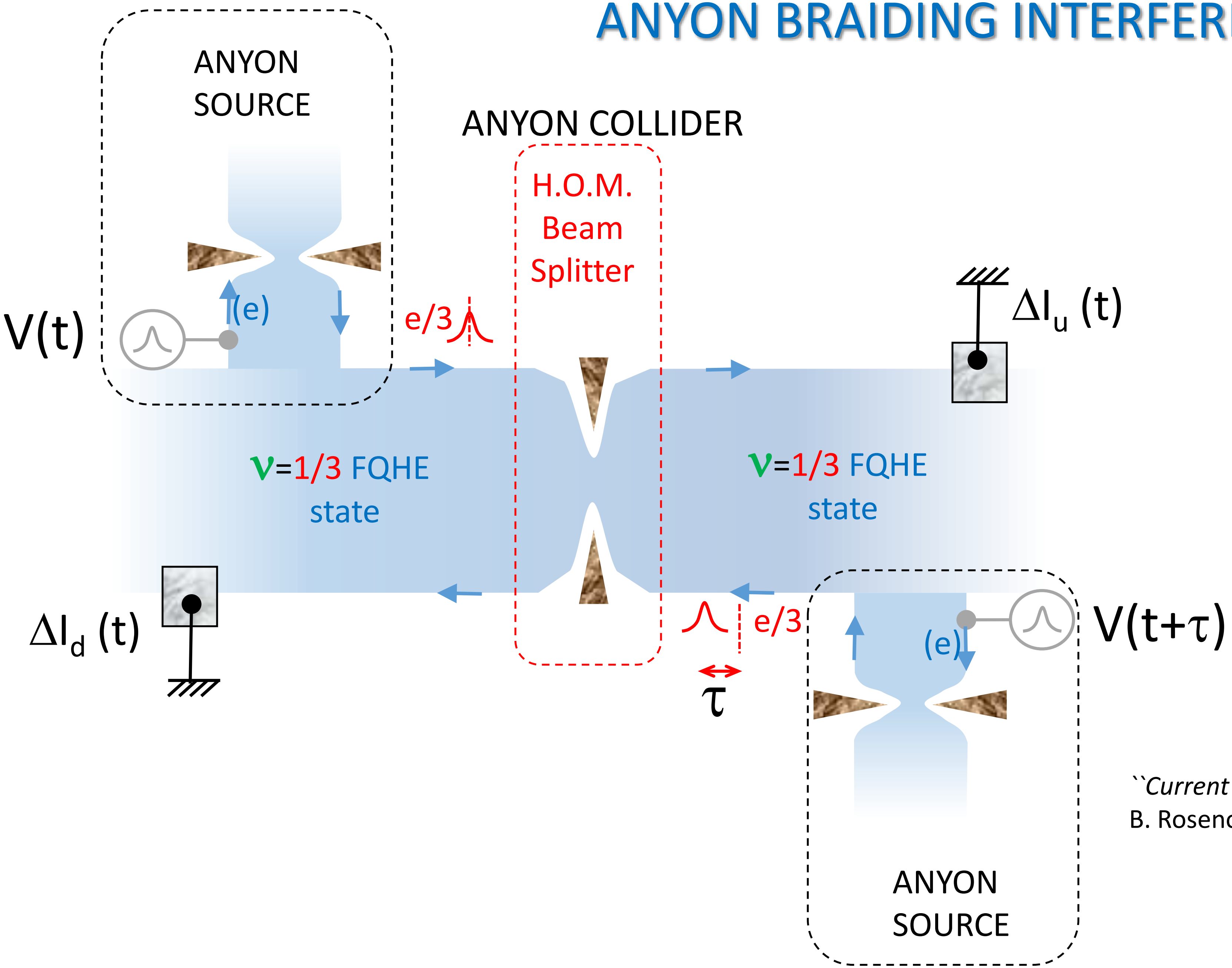
## A Time Controlled Poissonian Source of Anyon

IDEA: Weak backscattering beaks the leviton into e/3, 2e/3 quasiparticles.




- Anyons inherit from the time properties of Levitons
- Non-deterministic: Poissonian source

# PERSPECTIVE :
# ANYON BRAIDING INTERFERENCE

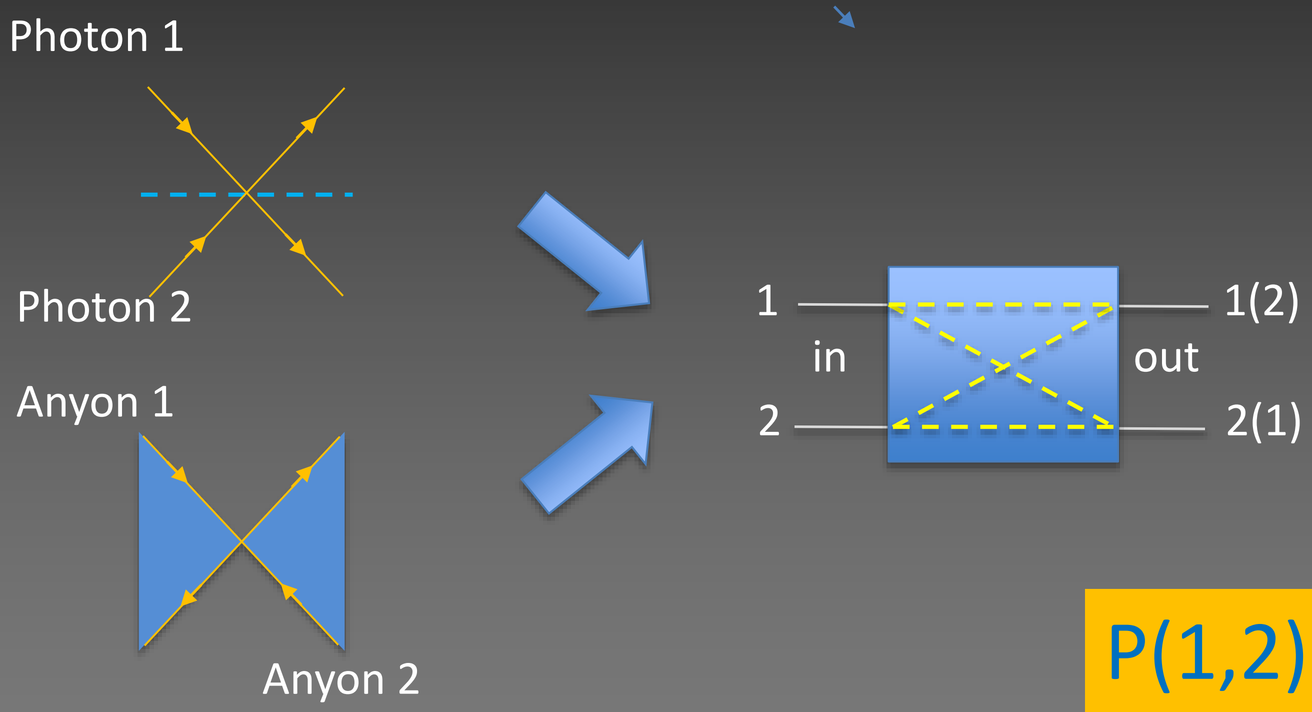



$$-\langle \Delta I_u \, \Delta I_d \rangle \propto 1 + g_2(\tau)\cos(\theta_{stat.})$$

*``Current Correlations from a Mesoscopic Anyon Collider ''*
B. Rosenow, I. P. Levkivskyi, B. I. Halperin, (2016)

# Braiding Anyons

## 1) Unveiling the anyon statistical angle with Hong Ou Mandel braiding interference




$P(1,2) = |b(1,2)+b(2,1)|^2$

$$P(1,2) = (1 - \cos\theta)/2$$

0 : boson bunching ($\theta$=0)

1 : fermion antibunching (($\theta=\pi$)

¼ : for $\nu$=1/3 FQHE abelian anyons ($\theta=\pi/3$)